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ORIGINAL ARTICLE

Enhancing brushless DC motor wheel design using single and multi-objective heat transfer search optimization approach

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Abstract

This study introduces and explores a groundbreaking single objective Heat Transfer Search (HTS) algorithm and decomposition-oriented Multi-Objective Heat Transfer Search (MOHTS/D) algorithm, specifically devised to address intricate issues in designing brushless direct current wheel motors in real-world scenarios. Drawing upon the principles established in the recently conceptualized thermodynamics laws grounded Heat Transfer Search algorithm, it harmonizes the conduction, convection and radiation stages to maintain a stable equilibrium between local intensification and global diversification during the search process. The aim is to pinpoint Pareto optimal solutions while verifying their encompassing characteristics through the methodical application of uniform weight vector sorting and Euclidean distance tactics within the conceived posteriori technique. To sidestep challenges such as local optimum confinement, heightened computational intricacy and reliability deficiencies, a decomposition-oriented strategy is embraced. The efficacy of the HTS and MOHTS/D algorithms has been scrutinized by subjecting it to rigorous analysis across Brushless Direct Current Motors (BLDC). Findings emphasize the formidable potential of HTS and MOHTS/D as a resilient optimization tool when measured against other recognized optimizers in the sphere of tangible, significant brushless direct current wheel motor design challenges. Notably, HTS and MOHTS/D demonstrates superior prowess in identifying optimal solutions across various Pareto fronts, surpassing other notable algorithms in this domain.

Keywords Global optimization · Pareto front · Brushless direct current wheel motor design · Euclidean distance · Weight vectors sorting · Metaheuristics · Constraints problems

1 Introduction

Multi-objective (MO) design optimization challenges markedly diverge from single-objective issues, given that

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MO scenarios entail the concurrent optimization of numerous distinct objectives. Furthermore, whereas single-objective problems tend to have a singular optimal solution, typically referred to as the global maximum or minimum, MO design predicaments offer a spectrum of potential solutions. These are predominantly classified as Pareto-optimal or non-dominated solutions, standing out as superior options within the specified search domain when all objectives are optimized in tandem [1]. However, these solutions might be less advantageous when considering one or more specific objectives, thereby categorizing the other available solutions as dominated. Due to the intrinsic non-dominance characteristic of the Pareto group in MO scenarios, each solution maintains a level of adequacy. Consequently, designers have the flexibility to select varying solutions based on their individual design needs or environmental factors. Therefore, understanding the diverse range of Pareto-optimal solutions becomes a crucial aspect in addressing MO optimization dilemmas [2].

Traditional approaches to tackling the complex, multi-modal and multi-constrained MO optimization dilemmas often falter and require a significant amount of computational resources. This downfall is primarily attributed to the conventional strategy these methods employ, which involves condensing multiple objectives into a singular focus through the utilization of weighted factor averages and highlighting a single Pareto-optimal solution at a time [3]. Optimization forms a cornerstone in the realm of engineering design. Consequently, amidst a plethora of real-world intricate issues with varied frameworks, metaheuristics (MHs) have emerged as a highly regarded instrument for optimization tasks. Owing to their simplicity, adaptability, absence of derivation constraints and capacity to evade local optimums, MHs have garnered immense attention, fostering the creation of celebrated techniques for multi-objective design dilemmas, such as MO Genetic Algorithm (GA) [4], MO Simulated Annealing (SA) [5] and others, including MO Differential Evolution (DE) [6] Though the solutions yielded by MHs aren't always optimal, they are achieved within a reasonable timeframe, with the distinctive ability to approach the Pareto front in a singular attempt being a standout feature of these MHs [7]. MO Particle Swarm Optimization (PSO) [8], alongside renowned algorithms like NSGA-II [3], MO Ant colony [9]. MOSOS algorithm [10] and SPEA2 [11] are used to find out optimality set of problems. However, aligning with the well-known 'No Free Lunch' theorem [12], it is acknowledged that no MH can address every problem both effectively and efficiently. An MH might prove beneficial for one specific design challenge, yet may falter when applied to another. In essence, no singular MH can guarantee the optimal solution across all problem domains. For instance, Hertz and De Werra [13] emphasized the superiority of the Tabu Search (TS) over SA in graph coloring issues, while Kuik and others [14] observed the reverse in lot-sizing problems. Conversely, Lee and Kim [15] noted comparable efficiency levels between TS and SA in project scheduling scenarios. Yang [16] further highlighted the lack of a universally accepted methodology to compare the efficacy of various MHs, making the quest for evolving potent MHs an ongoing research area [17, 18]. Remarkably, Mernik and colleagues [19] identified several misunderstandings in comparing MHs and Crepinsek et al. [20] warned about the inherent challenges in facilitating a substantial comparison between different MHs.

One of the notable drawbacks of several MHs, including GAs and SA, is their tendency to converge at a slow pace, resulting in increased computational expenses. Furthermore, there exists a risk of these algorithms becoming entrapped in a local optimum, a phenomenon observed in strategies like PSO, TS, ABC and ACO. To navigate past these hurdles, there has been a significant surge in the development of innovative, combined, enhanced and advanced MHs, focusing on amalgamating their favorable features [21]. A few standout examples in this category are MO hybrid GA [22], MO hybrid NSGA-II [23], MO hybrid PSO [24], MO enhanced ACO [25], combined MO cuckoo search [26] and MO hybridized PVS [27], among others. Within the context of MHs, maintaining a fluid equilibrium between global diversification and local intensification is vital. Essentially, diversification is linked to the exploration of the search space, whereas intensification is centered on leveraging accumulated knowledge from previous searches. It is imperative to strike a balance between these two aspects; diversification facilitates the rapid pinpointing of regions within the search space that potentially harbor high-quality solutions, while intensification tends to concentrate efforts in areas previously explored or those lacking in premium solutions, minimizing time wastage [28, 29]. Presently, the focus shifts towards the discovery of even more efficacious strategies, propelling a noticeable increase in the creation of new hybridized MHs. Despite the deployment of numerous hybrid MHs over recent decades by different scholars to tackle engineering design optimization issues—-favoring a blend of global and local search approaches—the sector remains somewhat underexplored and demands further attention.

The recently devised Heat Transfer Search (HTS) algorithm [30] draws its inspiration from the natural tendency of systems to attain a state of thermal equilibrium, adhering to the principles of heat transfer and thermodynamics. This population-based optimization strategy has been extensively utilized in the analysis of a plethora of unimodal, multimodal, composite and hybrid functions, in addition to being applied to a wide array of real-world benchmarks. Due to attributes such as its simplicity, user-friendliness, limited tuning parameters and superior rate of divergence, the HTS has managed to eclipse other prevalent algorithms including PSO, GA, DE, ABC and TLBO, noted in various studies.

The efficacy of the HTS has been substantiated in various domains, including truss structure optimization [31], non-linear economic dispatch configuration [32], the design and refinement of heat exchangers [33], the fine-tuning of machining process parameters [34], combined heat and power economic dispatch dilemmas [35] and economic power generation planning [36]. Certain researchers have ventured to enhance its functionality, proposing iterations like the sub-population-based HTS modification for structural optimization [37], a discrete form of HTS to tackle the traveling salesman problem [38], concurrent HTS [39], dynamic optimization problem-solving quadratic interpolation-based HTS [40] and an upgraded version to address unconstrained design matters [41].

Recognizing the pivotal role of MO optimization in tackling real-world design complexities, several studies have embarked on examining the versatility of HTS in this realm. For instance, Savsani et al. [42] explored an innovative MO variant of HTS, named MOHTS, to unravel dynamic half car passive ride optimization challenges, registering superiority over the renowned NSGA-II algorithm, particularly regarding the diversity and quality of non-dominated Pareto fronts. Additionally, Tawhid and Savsani [43] scrutinized the ε-constraint MOHTS technique in optimizing various engineering designs, highlighting its augmented effectiveness relative to MOGA, MOPSO, MODE, NSGA-II, PAES and MOWCA. In a separate study, Raja et al. [44] applied MOHTS to fine-tune parameters in a conflicting objectives-oriented plate heat exchanger design task, documenting a considerable reduction in the heat transfer coefficient by nearly 93%, corroborating well with experimental findings. Tejani et al. [45] introduced a MOHTS methodology for structural optimization issues, demonstrating its superiority over MOAS, MOACS and MOSOS techniques. Furthermore, to augment MOHTS's capabilities, variations including modified MOHTS [46] and advanced MOHTS [47] have been documented in scholarly articles.

While the algorithm rooted in stochastic parameters offers a range of benefits, it also exhibits significant gaps in its construction [12]. Notably, a high propensity for exploration within these algorithms often curtails their exploitation capacity and vice versa. This necessitates a harmonious balance between global diversification and local intensification to foster optimal outcomes [27]. Despite HTS showcasing efficacy in tackling intricate engineering optimization challenges, it tends to gravitate towards local optimums, a downside that cannot be overlooked [37, 38]. A notable limitation is its performance during the iterative phase of population size selection, wherein both small and large population sizes can sporadically produce superior responses. Nevertheless, alterations to the core HTS algorithm have illustrated rapid convergence velocities, enhanced accuracy and heightened robustness [42–47]. Given its recent inception, the HTS remains somewhat unexplored, beckoning further research to fully elucidate its potential in the optimization domain. Additionally, the nascent stage of HTS sparks curiosity for uncovering amendments that could bolster its efficacy and operational performance.

Responding to these challenges and opportunities, this research endeavors to craft and scrutinize a pioneering optimizer labeled as the decomposition-based multi-objective heat transfer search (MOHTS/D) algorithm, setting its sights on addressing a variety of tangible structural optimization predicaments. The decomposition-centric approach, noted for its adaptability and relevance in real-world MO design scenarios, has garnered the interest of a growing number of researchers [48, 49]. Consequently, this investigation seeks to amalgamate the merits of decomposition strategies with HTS, aspiring to formulate a resilient global optimization method adept at balancing local intensification and global diversification during the search process.

The research gap can be defined as the area where existing methodologies or algorithms fall short in addressing specific challenges in the design and optimization of brushless direct current (BLDC) wheel motors. Despite significant advancements in optimization algorithms for engineering applications, there remains a lack of algorithms that effectively balance the trade-off between global diversification and local intensification, particularly in complex, multi-objective optimization problems like BLDC motor design [50–53]. Previous methods may not fully capture the intricacies of thermodynamic principles in motor design or may suffer from slow convergence rates and susceptibility to local optima. The motivation behind this study stems from the urgent need to improve the efficiency and reduce the mass of BLDC motors, crucial for a wide range of applications, from electric vehicles to industrial machinery. The hunt for more sustainable and efficient electrical machines justifies the necessity for innovative optimization strategies that can address the multi-faceted objectives involved in motor design. This study is motivated by the potential to significantly enhance motor performance through a novel algorithm that leverages thermodynamics-based principles for optimization, filling a critical gap in existing engineering optimization practices.

The contributions of the forthcoming study are delineated as follows:

- Introduction and exploration of an innovative decomposition-based MO global optimization algorithm, leveraging evenly generated weight vectors sorting coupled with the Euclidean distance methodology.
- Aligning with the foundational principles of the HTS algorithm, the newly conceived MOHTS/D algorithm employs

phases of conduction, convection and radiation to foster a synergy between search intensification and diversification.

- Subjecting the MOHTS/D algorithm to testing through practical applications, such as the design challenges associated with brushless direct current wheel motors.
- Engaging in a qualitative and quantitative comparison of the MOHTS/D algorithm against four contemporary, leading-edge metaheuristic approaches. The derived results have promising applications in the development of more efficient and lighter electric vehicles, significantly contributing to the reduction of energy consumption and greenhouse gas emissions. In industrial applications, optimized BLDC motors can lead to substantial energy savings and operational efficiency, especially in automated machinery and robotics. In renewable energy systems, such as wind turbines or solar trackers, the application of these optimization results can enhance the reliability and performance of the systems, further promoting the adoption of sustainable energy sources.

The structure of the upcoming document is organized as follows: Sect. 2 is problem formulation. Section 3 delineates the underlying principles of the basic HTS algorithm with the proposed decomposition-oriented MOHTS approach and its operational dynamics; Sect. 4 is fuzzy decision making approach that is outlines the mathematical models pertinent to the considered MO design optimization, followed by an exhaustive discussion and analysis of the results derived from all test instances in Sect. 5; Finally, Sect. 6 encapsulates the principal conclusions drawn from the study, supported by performance metrics and derived Pareto fronts, along with insights into future prospects.

2 Problem formulation

The primary objective of BLDC motor [54] is to heighten efficiency by fine-tuning the following critical factors: peak current I_{max}, aggregate mass M_{tot}, ambient temperature T_a, interior diameter D_{int} and external diameter D_{ext}. The aggregate mass M_{tot} is identified as a criterion for reduction, forming the crux of a multi-faceted optimization challenge pertaining to a BLDC motor with several objectives. To facilitate the flawless integration of the motor into a wheel rim without inducing demagnetization in the magnet, adherence to specific parameters is imperative. The external diameter must be confined to under 340 mm, concurrently, the peak current I_{max} should align with 125 A and the aggregate mass M_{tot} must not surpass 15 kg.

Furthermore, due to mechanical requisites, the interior diameter D_{int} needs to be greater than 76 mm, aligning with five times the full load current. This requirement originates from the assorted mechanical limitations inherent to the motor's architecture. Consequently, the BLDC motor's developmental phase demands optimization across five design variables: B_e, D_s, B_d, B_{cs} and ζ. In parallel, the adherence to six inequality constraints is mandatory, encompassing the rotor's length proportion to a single stator component (Rrs), the magnetic length of the motor (Lm), the air-gap (e), input voltage (Vdc), the number of pole pairings (P) and the mean magnetic induction present in the rotor's yoke (Bcr). Adhering to these standards guarantees the motor's optimal operation, striking a harmonious equilibrium between performance attributes and physical limitations.

The guiding equation utilized to steer the optimization trajectory is delineated in Eq. (1). This formula acts as the fundamental mathematical blueprint, aiding in maneuvering the intricate interactions between the aforementioned parameters and limitations throughout the engineering and optimization stages. This complex endeavor is central to fabricating a BLDC motor that is both proficient and aptly tailored for its intended application.

$$\left.\begin{array}{c} \text{Maximize} \rightarrow f_1(\eta) \\ \text{Minimize} \rightarrow f_2(M_{tot}) \\ M_{tot} \leq 15Kg,\ D_{ext} \leq 340mm \\ D_{int} \geq 76mm,\ I_{max} \geq 125A \\ T_a \leq 120^{\circ}C,\ discr(D_s, B_d, B_e, \zeta) \geq 0 \\ 150mm \leq D_s \leq 330mm,\ 0.5T \leq B_e \leq 0.76T \\ 2A/\text{mm}^2 \leq \zeta \leq 5A/\text{mm}^2,\ 0.9T \leq B_d \leq 1.8T \\ 0.6T \leq B_{cs} \leq 1.6T \end{array}\right\} \quad (1)$$

Motor efficiency, denoted as η, is a measure of the effectiveness with which an electric motor converts electrical energy into mechanical energy. Specifically, it is defined as the ratio of mechanical power output to the electrical power input. This efficiency metric is crucial in the design of brushless DC (BLDC) motors as it directly impacts energy consumption and overall performance. In our optimization framework, motor efficiency is evaluated based on several design variables that influence both the numerator and denominator of the efficiency formula. These are magnetic flux density in the stator (B_e), the electric current density (ζ) and the dimensions of the motor, which affect the resistance and inductance, hence impacting the overall power losses. The optimization seeks to enhance η by minimizing these losses while maximizing the mechanical output.

- *B_e(Magnetic Flux Density in Stator)* This variable represents the magnetic flux density in the stator of the motor. It is crucial for determining the electromagnetic torque and thus directly impacts the motor's efficiency. The optimization seeks to maintain B_e within a specific range to ensure optimal magnetic properties while avoiding saturation of the motor components.

- D_s*(Stator Outer Diameter)* D_s is the outer diameter of the stator. This dimension is critical as it affects the overall size of the motor and its compatibility with the wheel rim. Proper sizing of D_s is essential for mechanical stability and to maintain the required air-gap between the stator and rotor.
- B_d*(Magnetic Flux Density in Rotor)* Similar to B_e, B_d pertains to the magnetic flux density, but in the rotor. Optimizing B_d influences the motor's output power and heat dissipation characteristics. It is adjusted to enhance performance while preventing demagnetization of the rotor magnets.
- B_{cs}*(Back Core Saturation)* B_{cs} refers to the saturation level of the back core of the stator. Managing this variable helps in optimizing the magnetic circuit of the motor, crucial for minimizing energy losses and enhancing efficiency.
- ζ*(Current Density)* ζ is the current density within the motor windings. It is a pivotal factor in determining the thermal and electrical loading conditions of the motor. By optimizing ζ, the motor can achieve higher efficiency and reliability by balancing the thermal rise against performance criteria.

It is evident from Eq. (1) that two objectives are highlighted: minimizing the overall mass (*f2* (M_{tot})) and maximizing the motor efficiency (*f1*(η)), with the restriction that M_{tot}=15 kg.

3 Heat transfer search

3.1 Classical HTS algorithm

The foundational concept behind the Heat Transfer Search (HTS) algorithm stems from the thermodynamic principle of achieving thermal equilibrium between a system and its environment [30]. Within the realm of thermodynamics, a system is perpetually driven to attain a state of energy balance with its surroundings, especially when the molecules are situated in a state of elevated or diminished energy. To facilitate this stability, three primary heat transfer mechanisms are employed, namely, conduction, convection and radiation. The algorithm has been devised to ensure equal likelihood for the operation of these three thermal transfer phases. Additionally, to foster a symbiotic relationship between local intensification and global diversification, the algorithm's search procedure has been adapted. Specifically, throughout the initial stages (the first half of the process), all three heat transfer phases collaboratively facilitate global diversification within the search space, which transitions into a focus on local intensification during the latter stages, as triggered by a predetermined number of generations.

In a manner akin to other optimization techniques inspired by natural phenomena, the HTS initiates its search journey through the stochastic generation of a population, consisting of 'n' members and 'm' design variables, representative of molecules at diverse temperature states. The objective fitness values bear resemblance to molecular energy tiers, with the optimal solution portraying the surrounding environment. Operating on the principles of conduction, convection and radiation during each iteration (Iter = 1, 2, 3…, Itermax), following the initial population setup, there is a continual adaptation of the population. The HTS adopts a greedy selection strategy, wherein a solution that surpasses its predecessor is retained for the subsequent iteration. Conversely, subpar solutions are supplanted by the most proficient solutions and in cases where duplicate solutions emerge, they are substituted with newly generated random solutions. The HTS's three heat transfer phases are—

3.1.1 Conduction phase

This stage is orchestrated by the Fourier's principle of thermal conduction, which posits that molecules possessing greater energy will impart heat to those at a lower energy tier, aspiring to reach a state of equilibrium. Through the tangible interactions among the molecules within the system and its environs, heat transfer through conduction becomes a feasible occurrence. During this stage, population alterations are directed by Eqs. 2 and 3, as delineated below:

$$S_{p,i}^{new} = \begin{cases} S_{q,i}^{old}\left(1 - R1^2\right), iff(S_p) > f(S_q) \\ S_{p,i}^{old}\left(1 - R1^2\right), iff(S_p) < f(S_q) \end{cases}; \quad ifIter \le Iter_{max}/F_{cond} \tag{2}$$

$$S_{p,i}^{new} = \begin{cases} S_{q,i}^{old}\left(1 - r_i\right), iff(S_p) > f(S_q) \\ S_{p,i}^{old}\left(1 - r_i\right), iff(S_p) < f(S_q) \end{cases}; \quad ifIter > Iter_{max}/F_{cond} \tag{3}$$

In this context, p and q signify random solutions with $p \neq q$ and can assume any value within the range of $porq \in$ (1,2, ..., n);i denotes a random variable index, falling within the scope of $i \in$ (1,2, ..., m) both R1, restricted to the interval $R1 \in$ [0, 0.3333] and r_i, confined within $r_i \in$ [0,1], serve as the probability value and a random number pertaining to the conduction phase, respectively. The conduction coefficient, F_{cond}, is allocated a value of 2 for the initial half of the entire generation span [37, 41, 42]. In this segment, the two functions are analyzed in contrast and the less viable solution undergoes substitution as per Eq. 2, whereas Eq. 3 governs the replacement process in the search's subsequent half, fostering a blend of local intensification and global diversification in the algorithm, aiding in pinpointing the optimal solution.

3.1.2 Convection phase

Transitioning to the next phase, which is regulated by the principles of convective heat transfer (or Newton's cooling law), the molecules within the system, having an average temperature denoted by S_{mean}, engage with the surrounding elements, characterized by a temperature of S_{surr}, in a quest to achieve thermal equilibrium. The latter entity is recognized as the superior solution in this context. Updates to the solutions follow a distinct pattern, as noted below:

$$S_{p,i}^{new} = S_{p,i}^{old} + R2 \times [S_{surr} - S_{mean} \times abs(R - r_i)] \quad if Iter \leq Iter_{max}/F_{conv} \tag{4}$$

$$S_{p,i}^{new} = S_{p,i}^{old} + R2 \times [S_{surr} - S_{mean} \times round(1 + r_i)] \quad if Iter > Iter_{max}/F_{conv} \tag{5}$$

It is vital to highlight that each design variable undergoes transformations throughout this phase, aligned with the alterations to solution S_p. The value of R2 lies between [0.6666, 1] and the convective factor, F_{conv}, is established at 10 [45–47]. Both Eqs. 4 and 5 play a pivotal role in administering both local intensification and global diversification strategies.

3.1.3 Thermal radiation phase

The final stage embraces the radiation mode as the cornerstone heat transfer mechanism, seeking to rectify the thermal disparities between the system and its surroundings. It is noteworthy that exchanges can transpire either internally among system molecules or externally between system molecules and those in the surrounding environment. Following the directives of Eqs. 6 and 7, the solutions undergo the necessary updates, as illustrated below:

$$S_{p,i}^{new} = \begin{cases} S_{p,i}^{old}(1 - R3) + R\left(S_{q,i}^{old}\right), if f(S_p) > f(S_q) \\ S_{p,i}^{old}(1 + R3) - R\left(S_{q,i}^{old}\right), if f(S_p) < f(S_q) \end{cases}; \quad if Iter \leq Iter_{max}/F_{rad} \tag{6}$$

$$S_{p,i}^{new} = \begin{cases} S_{p,i}^{old}(1 - r_i) + r_i\left(S_{q,i}^{old}\right), if f(S_p) > f(S_q) \\ S_{p,i}^{old}(1 + r_i) - r_i\left(S_{q,i}^{old}\right), if f(S_p) < f(S_q) \end{cases}; \quad if Iter > Iter_{max}/F_{rad} \tag{7}$$

The probability variable R3 finds its place in the range [0.3333, 0.6666] and F_rad, the radiation coefficient, maintains a fixed position at 2 [37, 42, 45–47]. An encompassing visualization of the operational facets of all three phases of the HTS is encapsulated in Fig. 1.

3.2 MOHTS/D algorithm

The decomposition-centric multi-objective optimization algorithm was pioneered by Zhang and Li in 2007 [55]. This paper presents an innovative version of this algorithm, integrated with HTS, termed Multi-Objective Heat Transfer Search based on Decomposition (MOHTS/D), specifically crafted to address structural optimization challenges. The MOHTS/D technique dissects Multi-Objective Problems (MOPs) into a series of scalar optimization sub-tasks. These sub-tasks employ decomposition methods, optimizing them concurrently using the HTS approach.

In this structure, the MOHTS/D fragmentizes the Pareto Front (PF) approximation into multiple scalar optimization sub-tasks, utilizing an array of evenly distributed weight vectors $\lambda^m = (\lambda_1^m, \lambda_2^m)^T$ to dissect the MOPs. Within the context of BLDC design optimization, N is perceived with each weight vector embodying a dual-dimensional aspect for the pair of objectives, denoted as $\lambda^m = (\lambda_1^m, \lambda_2^m)^T$ Here, m ranges from 1 to N with each m corresponding to a specific sub-task. The sum $\lambda_1^m + \lambda_2^m = 1$ ensures that each weight vector is properly normalized. The superscript 'm' denotes the index of a sub-task or component within the decomposition framework of the MOHTS/D algorithm. Specifically, it identifies individual scalar optimization tasks derived from the original multi-objective problem (MOP). It is hypothesized that $z^* = (z_1^*, z_2^*)^T$ represents the vector housing the minimal objective values, serving as a reference point. The components of this vector are derived as $z_1^* = \min\{f_1(x)|x \in \Omega\}$ and $z_2^* = \min\{f_2(x)|x \in \Omega\}$. Subsequent to the division of the MOP into N sub-issues, the objective function of the m-th sub-issue is expressed as:

$$g^{te}(x|\lambda^m, z^*) = \max\left\{ \begin{matrix} \lambda_1^m |f_1(x) - z_1^*| \\ \lambda_2^m |f_2(x) - z_2^*| \end{matrix} \right\} \tag{8}$$

It is plausible that z* may not be determined in advance. As the search progresses, the algorithm employs the minimum values discovered for $f_1(x)$ and $f_2(x)$ thus far to replace z_1^* and z_2^*, respectively, in the subproblem objectives. MOHTS/D orchestrates a synchronous minimization of all N objective functions (for N sub-tasks) in a single execution, with each sub-task being optimized through insights primarily gleaned from neighboring sub-tasks. The MOHTS/D ensures an equitable distribution of computational efforts across all sub-tasks.

Throughout the search, MOHTS/D adopts the Tchebycheff method to sustain:

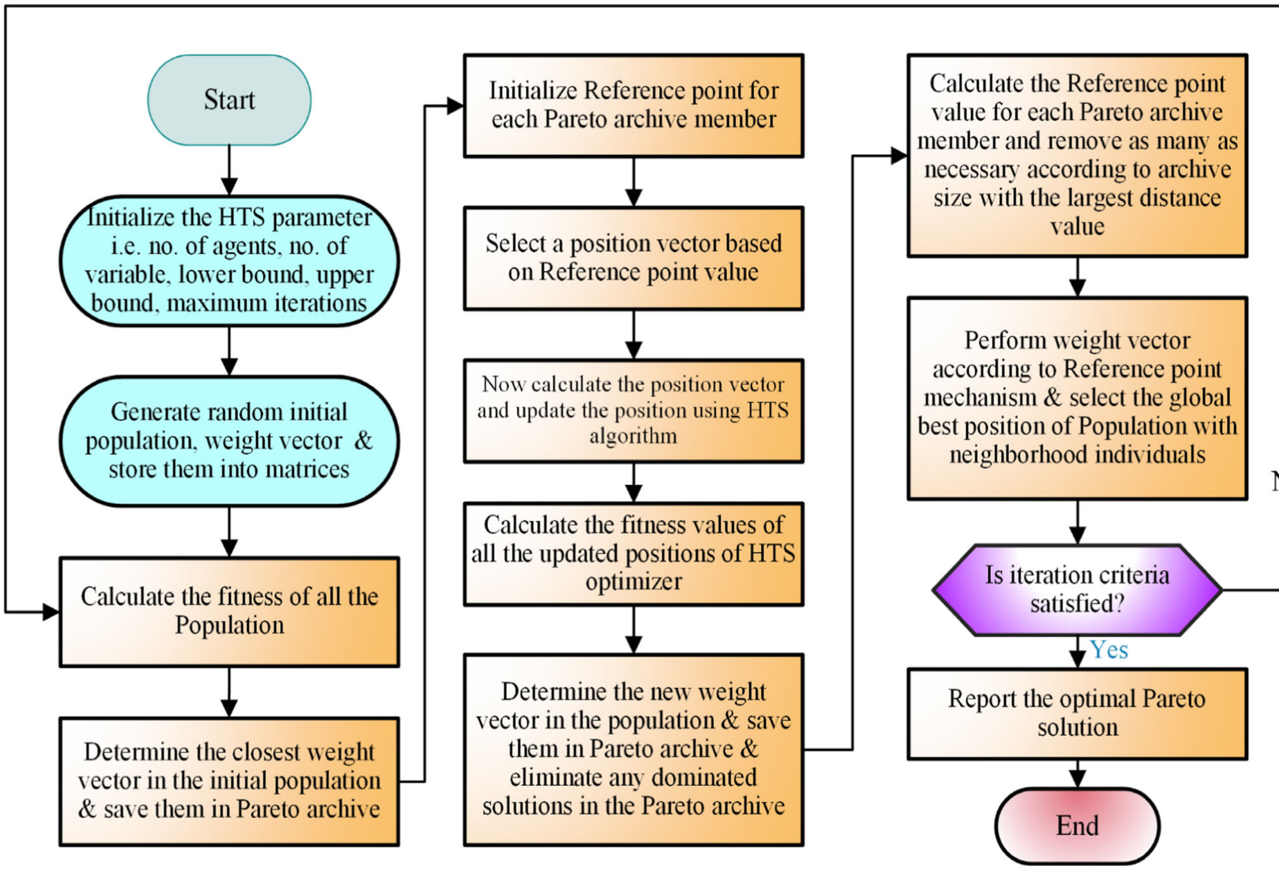

Fig. 1 Flowchart of the MOHTS/D algorithm

- A compilation of N vectors $x^1, x^2, \ldots., x^N$, with each vector in this particular issue encompassing 100 elements.
- Function values documented as $FV^1, FV^2, \ldots., FV^N$ where $FV^m = \{f_1(x^m), f_2(x^m)\}$ for $m = 1,2, \ldots., N$.
- $z^* = (z_1^*, z_2^*)^T$, with z_1^* and z_2^* representing the lowest values ascertained so far for evaluating $f_1(x)$ and $f_2(x)$.
- $z^{nad} = (z_1^{nad}, z_2^{nad})^T$ where z_1^{nad} and z_2^{nad} are the peak values identified to date while assessing functions $f_1(x)$ and $f_2(x)$.

The general framework of MOHTS/D is presented in Algorithm 1.

Algorithm 1 MOHTS/D Multi-objective Optimization

Input:

- A multi-objective optimization problem, represented as: min $F(x) = [f_1(x), \dots, f_m(x)]^T$ subject to x in Ω
- A collection of N weight vectors: $w^i = (w_1^i, \dots, w_m^i)^T$, for $i = 1$ to N
- Termination criteria
- N: count of sub-problems
- T_m: neighborhood population dimension
- $MaxIter$: maximum allowed iterations
- $EvalCount = 0$: counter for function evaluations

Procedure:

1. Setup

1. Determine the T_m nearest weight vectors for each weight vector using the Euclidean distance. For every sub-problem i (from 1 to N), initialize $\text{pop}^i = \{i_1, \dots, i_{T_m}\}$, where $w^{i_1}, \dots, w^{i_{T_m}}$ are the nearest weight vectors to w^i.
2. Construct an initial population $P = \{x^1, \dots, x^N\}$ by uniformly selecting from Pareto optimal solutions (Ω). Compute the fitness value FV^i for each solution x^i as $FV^i = (f_1(x), \dots, f_m(x))$. Set $FV^i = FV^1(x^1), \dots, FV^N(x^N)$.
3. Set the ideal point $z^* = (z_1^*, \dots, z_m^*)^T$ with $z_j^* = \min\{f_j(x) | x \in \Omega, j = 1, \dots, m\}$ and the nadir point $z^{nad} = (z_1^{nad}, \dots, z_m^{nad})^T$ with $z_j^{nad} = \max\{f_j(x) | x \in \Omega, j = 1, \dots, m\}$.
4. Increment $EvalCount$ by N and set generation count $gen = 1$.

2. Iteration

while termination criteria are not met:

for i from 1 to N:

a. Reproduction: Generate a solution y' using the HTS method and then derive a solution y from y'.

b. Repair: If any component of y is outside Ω, regenerate its value within Ω.

c. Compute the fitness value for the new solution y.

d. Update Ideal and Nadir Points: For j from 1 to m, if $z_j^* \leq f_j(x^i)$ then set $z_j^* = f_j(x^i)$. If $z_j^{nad} \geq f_j(x^i)$ then set $z_j^{nad} = f_j(x^i)$.

Increment $EvalCount$ by N and gen by 1.

3. Termination Check

if termination criteria are met or $MaxIter$ is achieved:

Exit the loop.

Output:

- Pareto set $PS = \{x^1, \dots, x^N\}$
- Pareto front $PF = \{FV^1(x^1), \dots, FV^N(x^N)\}$

The flowchart of the MOHTS/D algorithm is presented in Fig. 1.

4 Fuzzy decision making based optimal solution section

To ascertain the most appropriate and optimum solution from the obtained Pareto-optimal set, this research adopts a fuzzy-based compromise solution methodology [56]. This approach seeks a solution that delivers the highest level of satisfaction for every objective, drawn from the pool of potential Pareto-optimal outcomes and is guided by fuzzy membership functions. Given the potential vagueness that decision-makers might encounter, each optimization query pertaining to the jth solution is characterized by a membership function denoted as μ_i^j. It is hypothesized that μ_i^j represents a monotonic function, structured as detailed below:

$$\mu_i^j = \begin{cases} 1, \; iff_i^j \, f_{\min}^j \\ \frac{f_{\max}^j - f_i^j}{f_{\max}^j - f_{\min}^j}, \; iff_{\min}^j \, f_i^j \, f_{\max}^j \\ 0, \; iff_i^j \, f_{\max}^j \end{cases} \tag{9}$$

Constructing the normalized membership function at every non-dominated solution follows the subsequent methodology:

$$\mu_i = \frac{\sum_{j=1}^{N_{obj}} \mu_{ij}}{\sum_{i=1}^{M} \sum_{j=1}^{N_{obj}} \mu_{ij}} \tag{10}$$

Here, M symbolizes the quantity of non-dominated solutions, N_{obj} stands for the count of objective functions and $f_{\max}^j$ and $f_{\min}^j$ are representative of the maximum and minimum values of the pertinent objective function, respectively. A solution is considered to be the superior compromise when it exhibits a high μ_i value.

5 Results and discussion

5.1 MOHTS/D algorithm results for brushless motor wheel design problem

Upon evaluating various test scenarios, this study embarked on the optimization of the BLDC motor structure utilizing four distinct multi-objective algorithms, including the newly introduced MOHTS/D technique. Given the presence of six specific constraints in the BLDC wheel motor design task, the new strategy operates synergistically with a constraint handling framework. This study employs a fuzzy membership method in tandem with the proposed algorithm to pinpoint BC results. Following the acquisition of Pareto solutions and identifying the BC within the spectrum of non-dominated solutions—a process fluidly adapting during the selection phase—it serves to formulate BC determinations concerning trade-off attributes. This segment deliberates on the results stemming from the application of the recommended algorithm, along with alternative chosen algorithms, in addressing the BLDC wheel motor design dilemma. A few of the algorithms being contemplated for implementation encompass HTS, ALO [57], IMO [58] and SCA [59]. To date, these algorithms have not been scrutinized in the context of BLDC wheel motor challenges. Prompt action is taken to deploy all selected algorithms to surmount this design hurdle, optimizing variables such as minimizing the motor's aggregate mass or amplifying its performance efficiency. As documented in Table 1, each of the objective functions and control parameters has been subjected to a ten-cycle analysis through the corresponding algorithms. The decision regarding the supreme control parameters is influenced by numerous trial operations and scholarly article analyses. Each of the triumphant algorithms adeptly navigated through complex engineering impediments. In a bid to broaden the analysis of the algorithm's effectiveness, the BLDC motor blueprint elucidated in Sect. 2 is incorporated. As noted earlier, every algorithm is executed tenfold for each pair of designated issues.

5.1.1 Case 1: Maximize efficiency of BLDC motor

In this analysis, the objective is to fine-tune the BLDC motor's design variables in line with the criteria of objective function 1, as delineated in Eq. (1). The algorithms chosen for this task, namely HTS, ALO, SCA and IMO, are directly engaged with objective function 1. The outcome of design variables from all 10 distinct evaluations is shown in Table 1. The highlighted figures in each table represent the optimal outcomes from the 10 analyses. According to Table 1**,** the pinnacle of motor efficiency was recorded by the HTS, ALO, SCA and IMO algorithms, registering figures of 95.329, 95.285, 95.083 and 95.300% respectively. The analysis reveals that HTS has the highest efficiency at 95.329%, with ALO, SCA and IMO showing lower efficiencies by 0.046, 0.258 and 0.030% respectively. These differences highlight the superior performance of HTS in maximizing efficiency shown in box plot Fig. 2.

5.1.2 Case 2: Minimize mass of BLDC motor

In a parallel analysis, the design variables of the BLDC motor are scrutinized with the guidelines of objective function 2, outlined in Eq. (2). Similar to the prior analysis, chosen algorithms like HTS, ALO, IMO and SCA are put to work directly

Table 1 Analysis of Single-Objective Optimization (Focusing on Maximizing Efficiency) in BLDC Wheel Motor Design

Control Variables	HTS	ALO	SCA	IMO
$B_d(T)$	1.800	1.789	1.800	1.797
$B_e(T)$	0.654	0.651	0.674	0.649
$B_{cs}(T)$	1.012	1.215	0.767	1.013
$D_s(mm)$	0.202	0.205	0.204	0.203
$\zeta(A/mm^2)$	2,016,111.418	2,000,000.000	2,569,922.823	2,029,918.503
Efficiency in % (η)	95.329	95.285	95.083	95.300

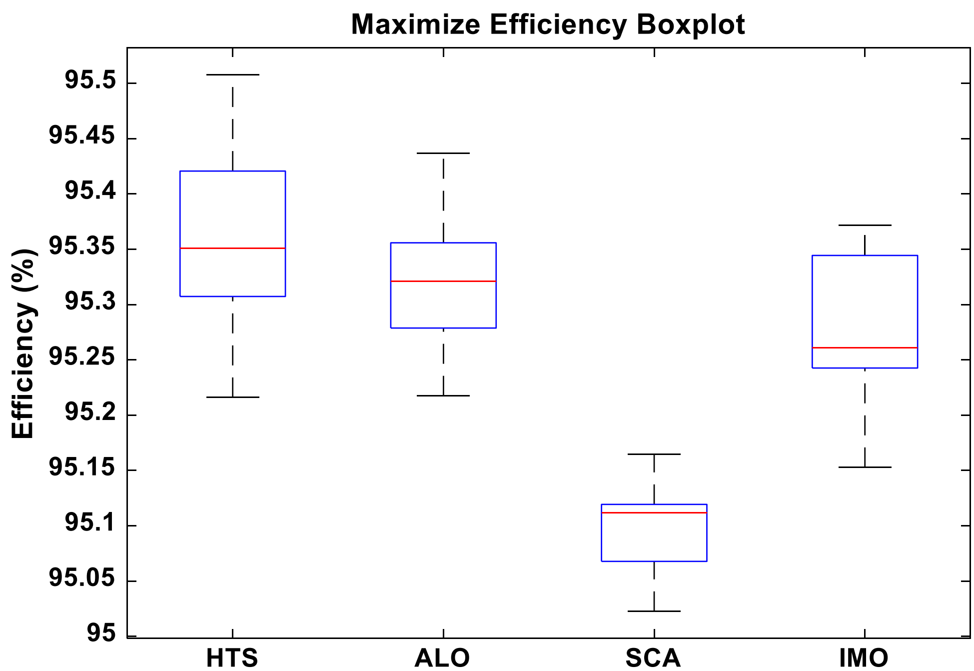

Fig. 2 Box plot for efficiency maximization using HTS, ALO, SCA and IMO algorithms

Table 2 Examination of Single-Objective Optimization (Aiming at Minimizing Total Mass) in BLDC Wheel Motor Design

Control Variables	HTS	ALO	SCA	IMO
$B_d(T)$	1.800	1.800	1.800	1.800
$B_e(T)$	0.652	0.649	0.649	0.659
$B_{cs}(T)$	1.600	1.599	1.600	1.585
$D_s(mm)$	0.189	0.193	0.198	0.188
$\zeta(A/mm^2)$	3,796,706.388	3,990,871.332	3,766,580.349	3,741,009.529
Total mass in Kg (M_{tot})	10.5689	10.5836	11.1067	10.6051

on objective function 2. The culmination of optimized design variables from ten separate runs are recorded in Table 2, with the prominent figures in each table signalling the best outcomes from the ten evaluations. As noted in Table 2, HTS managed to obtain the least motor mass, amounting to 10.5689 kg. Following closely were the figures from ALO, IMO and SCA, which reported masses of 10.5836, 11.1067 and 10.6051 kg, respectively. The analysis reveals that HTS has the lowest total mass at 10.5689 kg, with ALO, SCA and IMO showing higher total masses by 0.0147, 0.5378 and 0.0362 kg respectively. These differences highlight the superior performance of HTS in minimizing the total mass. Specifically, ALO total mass is approximately 0.139% higher than HTS, SCA total mass is approximately 5.089% higher than HTS and IMO total mass is approximately 0.343% higher than HTS. This demonstrates that HTS consistently outperforms the other algorithms in achieving the objective of minimizing the total mass in the BLDC wheel motor design. The graphical representation of the convergence trajectory for all chosen algorithms can be observed in Fig. 3. This

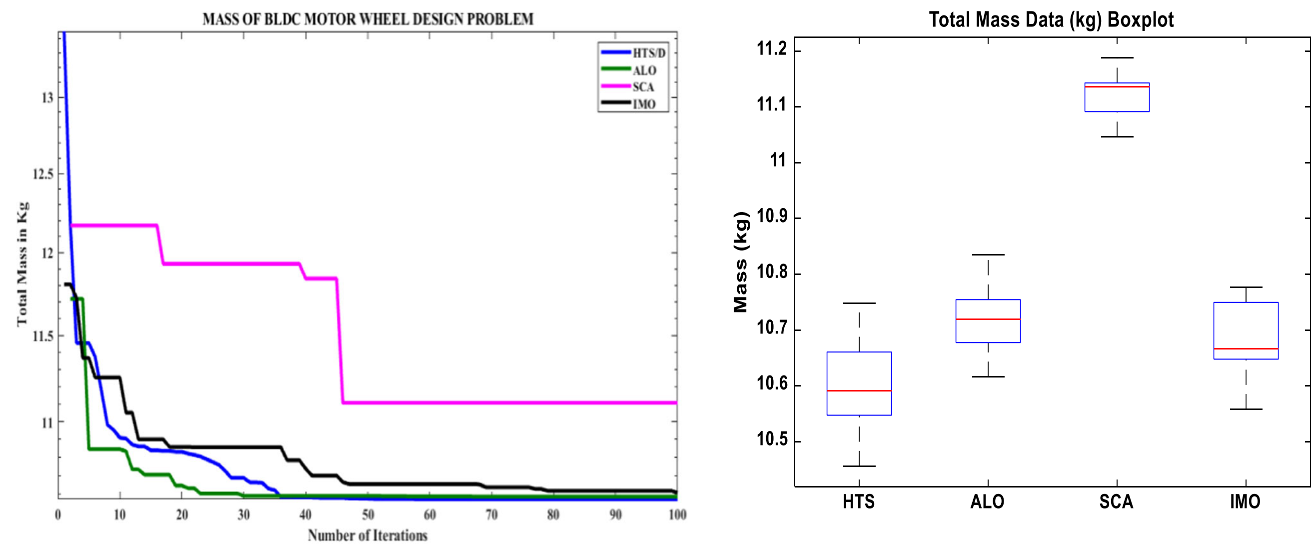

Fig. 3 Convergence curve and Box plot for efficiency maxmimization using HTS, ALO, SCA and IMO algorithms

Table 3 Comparative Analysis of Multi-Objective Optimization (Focusing on Maximizing Efficiency and Minimizing Total Mass) based on best compromise solution in BLDC Wheel Motor Design

Control Variables	MOHTS/D	MOTEO	MOEO	MOTLBO
$\boldsymbol{B_d}(T)$	1.732	1.773	1.683	1.785
$\boldsymbol{B_e}(T)$	0.657	0.683	0.654	0.664
$\boldsymbol{B_{cs}}(T)$	1.206	1.103	1.029	1.058
$\boldsymbol{D_s}(mm)$	0.197	0.197	0.200	0.195
$\boldsymbol{\zeta}(A/mm^2)$	2,304,832.384	2,400,568.859	2,239,507.957	2,285,050.596
Total mass in Kg ($\boldsymbol{M_{tot}}$)	13.877	13.972	14.573	13.930
Efficiency in % ($\boldsymbol{\eta}$)	95.08	95.08	95.19	95.12

figure accentuates that the HTS algorithm reaches the minimal mass considerably faster compared to the ALO, IMO and SCA algorithms.

5.1.3 Case 3: Pareto optimization for maximize efficiency and minimize mass of BLDC motor

This segment encapsulates a discussion on the simulated analysis of the innovative MOHTS/D algorithm and draws a comparison with other contenders like MOTEO [60], MOEO [61] and MOTLBO [62]. Adhering to identical parameters as the single-objective optimization problem (comprising 30 archive size, 100 uppermost iterations and a populace of 30 individuals), the multi-objective optimization issue is executed tenfold. Table 3 demonstrates the outcomes from the most viable compromise, accompanied by the more pragmatic options. Additionally, Table 3 delineates the objective function values, encapsulating aspects like motor efficiency, motor mass and a balanced compromise between the two, coupled with the five design/optimization variables—B_e, D_s, B_d, B_{cs} and ζ, which were fine-tuned using all four algorithms. The Mass analysis Table 3 reveals that MOHTS/D has the lowest total mass at 13.877 kg, with MOTEO, MOEO and MOTLBO showing higher total masses by 0.095, 0.696 and 0.053 kg respectively. These differences highlight the superior performance of MOHTS/D in minimizing the total mass. Specifically, MOTEO total mass is approximately 0.684% higher than MOHTS/D, MOEO's total mass is approximately 5.015% higher than MOHTS/D and MOTLBO total mass is approximately 0.383% higher than MOHTS/D. This demonstrates that MOHTS/D consistently outperforms the other algorithms in achieving the objective of minimizing the total mass in the BLDC wheel motor design. The efficiency analysis in Table 3 reveals that MOHTS/D achieves an efficiency of 95.08%, with MOTEO, MOEO and MOTLBO showing slightly higher efficiencies by 0.00, 0.11 and 0.04% respectively. These differences highlight the superior balance of MOHTS/D in optimizing both efficiency and total mass. Specifically, MOEO efficiency is approximately 0.116% higher than MOHTS/D and MOTLBO efficiency is approximately 0.042% higher than MOHTS/D. This demonstrates that MOHTS/D consistently provides a well-rounded optimization solution, effectively balancing the objectives of overall efficiency and total mass

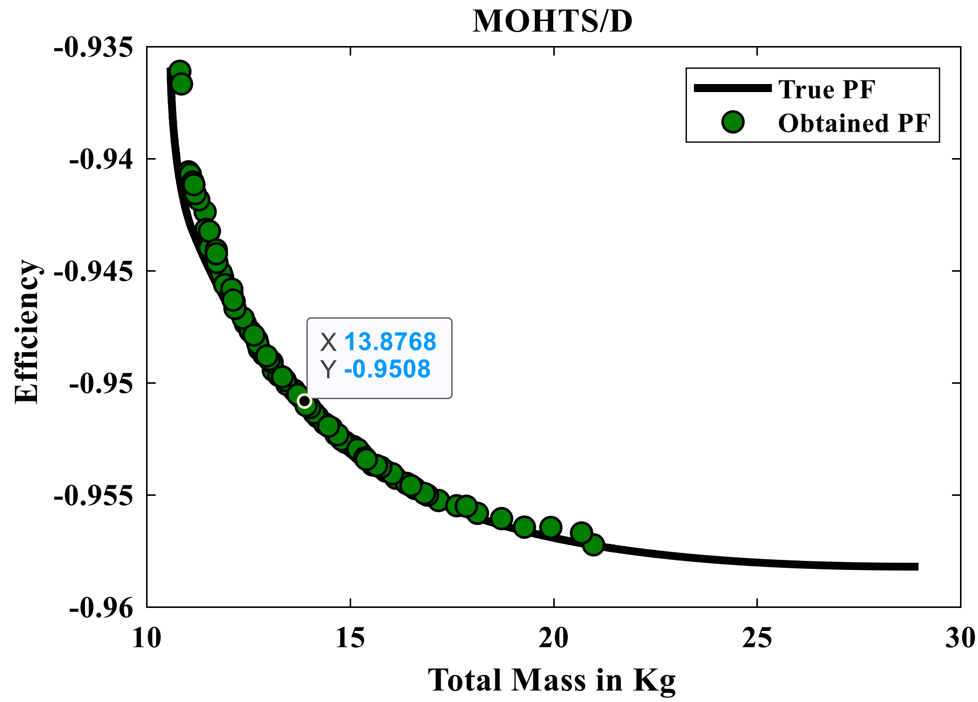

Fig. 4 Pareto optimal front obtained by MOHTS/D algorithm

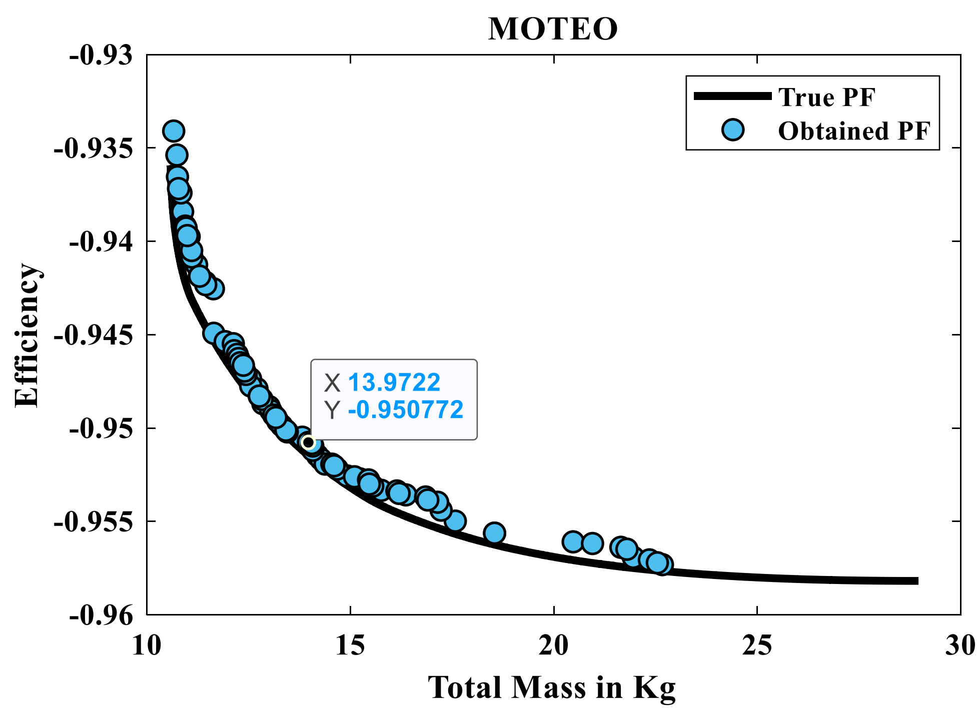

Fig. 5 Pareto optimal front obtained by MOTEO algorithm

in the BLDC wheel motor design. Figures 4, 5, 6 and 7 display the Pareto fronts for all the algorithms under scrutiny. To attain the apex of comparative Pareto solutions, as depicted in Fig. 8, it is imperative to fine-tune the decision variables. The recommended algorithm embodies a fixed penalty scheme, representing a proficient approach to maneuvering constraints. Preliminary observations denote that the newly proposed algorithm is on a trajectory of consistent progress, aiming to elevate both efficiency and motor mass metrics. These revelations substantiate the superior performance of the MOHTS/D algorithm over the rival options considered.

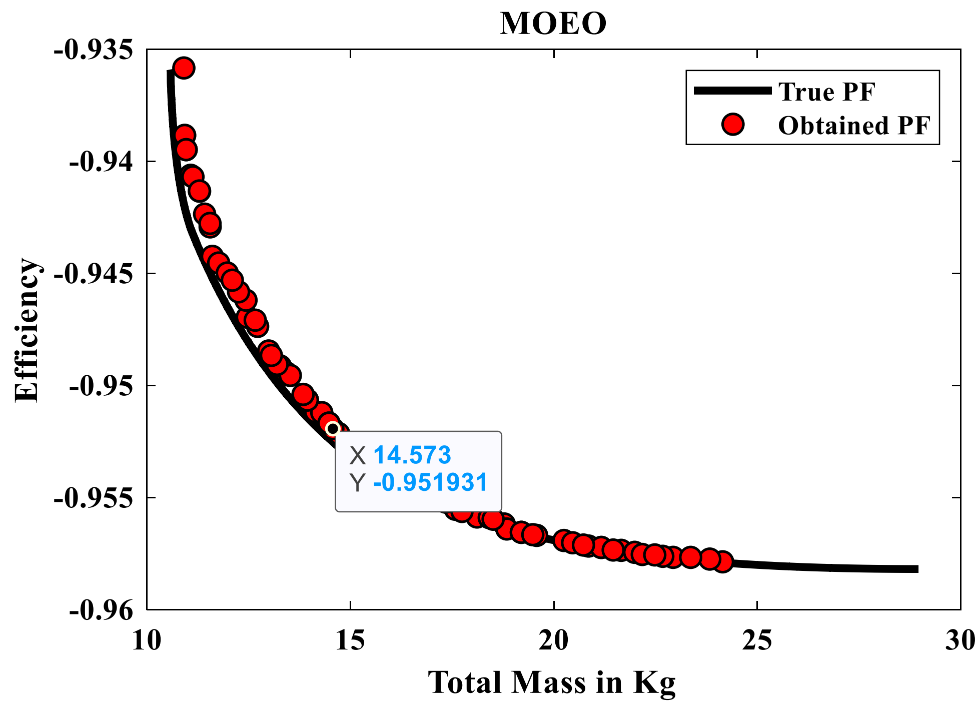

Fig. 6 Pareto optimal front obtained by MOEO algorithm

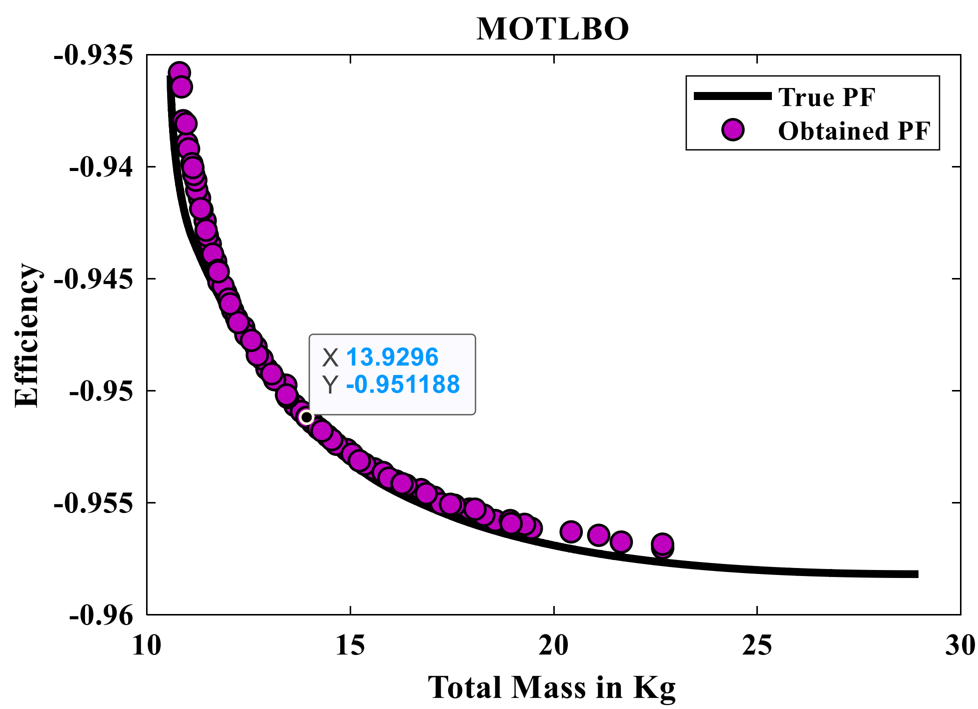

Fig. 7 Pareto optimal front obtained by MOTLBO algorithm

6 Conclusion

In this work, the enhancement of the brushless DC (BLDC) wheel motor design, focusing primarily on optimizing either the machine's efficiency or minimizing its overall mass is studied. Through an analytical framework informed by prior scholarly research, we employed a variety of single objective optimization strategies, including ALO, IMO and SCA. HTS stands out with the highest efficiency of 95.329%, significantly surpassing its competitors such as ALO, SCA and IMO by marginal but consistent percentages. Additionally, HTS proves to be the most effective in minimizing total mass, achieving a weight of 10.5689 kg, which is considerably lower than that of the other tested algorithms. This dual advantage highlights HTS capability in enhancing operational efficiency while reducing structural load.

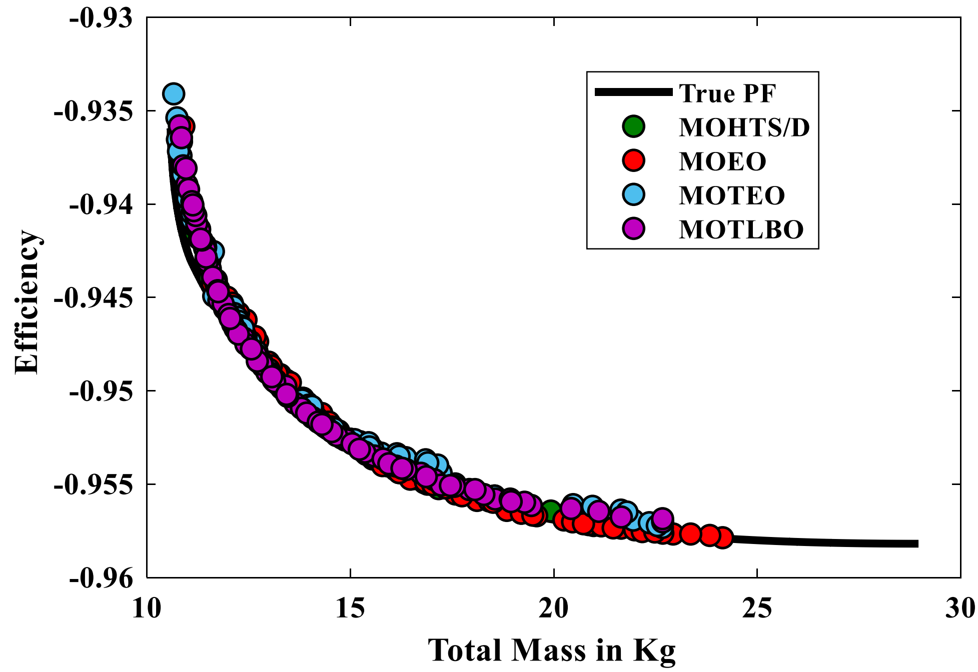

Fig. 8 Comparative Pareto front obtained by MOHTS/D, MOEO, MOTEO and MOTLBO algorithms

Furthermore, our exploration into multi-objective optimization, particularly with the MOHTS/D variant, has shown promising results in addressing the dual challenges of mass reduction and efficiency enhancement. MOHTS/D showcases its strength in achieving a well-balanced optimization between efficiency and mass reduction in the advanced models. It not only attains the lowest total mass at 13.877 kg when compared to MOTEO, MOEO and MOTLBO, but it also maintains competitive efficiency rates, narrowly trailing behind MOEO in terms of performance percentages. The results emphasize MOHTS/D exceptional ability to optimize key parameters effectively, providing a robust solution for engineering applications that demand precision in both efficiency and weight management. These findings affirm the critical role of algorithm selection in motor design, impacting both the efficiency and the physical attributes of the final product. The distinguished performance of HTS and MOHTS/D in our tests highlights their potential as preferred algorithms for future developments in motor technology, ensuring optimal performance and sustainability in design processes.

One limitation of this study is the reliance on the proposed MOHTS/D algorithm's performance under specific conditions and scenarios applied to BLDC motor design. The algorithm's generalizability to other types of motors or optimization problems in different domains may require further investigation. Additionally, while the decomposition-oriented approach offers substantial improvements, computational complexity and scalability to very large-scale problems might pose challenges. The effectiveness of the algorithm in real-world applications beyond the theoretical and controlled environments also warrants further exploration. Looking ahead, further research in this domain could potentially focus on integrating artificial intelligence and machine learning techniques to enhance the optimization process further.

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Data availability The data presented in this study are available through email upon request to the corresponding author.

Declarations

Conflict of interest The authors declare no conflicts of interest.

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