scientific reports

OPEN

Propagation of optical solitons and dispersive solitary wave structure in complex media to the nonlinear integrable system via computational technique

Mujahid Iqbal[1,2], Waqas Ali Faridi[3], Reem Algethamie[4], Abeer Aljohani[5], Mohammed El-Meligy[6,7], Nazar Mohammad[8]✉ & Khalid A. Alnowibet[9]

The nonlinear complex Shynaray-IIA (S-IIA) system is investigated in the present study. The main objective of this research to explore optical soliton solutions of the nonlinear complex Shynaray-IIA system using the auxiliary equation method. The obtained solutions exhibit novel and diverse physical structures, including periodic waves, peakon dark, bell bright, kink waves, peakon bright, anti-kink waves, bell dark, breather solitons, mixed bright and dark, mixed kink bright and dark, and mixed anti-kink bright and dark wave solitons. Furthermore, using Mathematica software, three-dimensional, two-dimensional, and contour plots have been generated to illustrate the physical behavior of the derived solutions by assigning appropriate constant parameters through symbolic computation. The obtained solutions are original, novel, and have not been previously examined for the Shynaray-IIA system. These results are expected to be useful for studying nonlinear phenomena in physical sciences and engineering. The proposed method is significant for addressing new problems and applying previously untested approaches, leading to the derivation of several exact and optical soliton solutions. The results demonstrate the simplicity, power, and efficiency of this technique for analyzing a variety of nonlinear problems in both real and complex forms.

Keywords Optical solitons, Nonlinear Shynaray-IIA system, Auxiliary equation method, Symbolic computation, Solitary wave structure, Analytical solutions.

Nonlinear equations play a vital role in engineering, science, and modern technology. Since linear theory has its limitations, it has become necessary to employ nonlinear analysis to study the dynamics of multi-scale and complex systems. Nonlinear partial differential equations (NLPDEs) form a fundamental part of nonlinear theory and have been widely used to investigate nonlinear phenomena in diverse fields such as quantum physics, chemistry, optical fibers, biology, nonlinear dynamics, computational fluid dynamics, ocean engineering, nonlinear optics, quantum mechanics, fluid mechanics, communication systems, and optical systems, among others[1–5]. It is not only important to understand the analysis of NLPDEs (including integrability analysis and the construction of exact solutions), but such analyses are also essential in many practical contexts. The extensive applications of nonlinearities across research communities highlight their significance and establish them as a crucial domain within the mathematical sciences. Consequently, in recent years, there has been a notable increase in research focused on exploring soliton solutions of NLPDEs. Whether a soliton is temporal or spatial depends on whether the localization of light occurs in time or in space during wave propagation.

[1]College of Information Science and Technology, Dalian Maritime University, Dalian 116026, Liaoning, PR China. [2]Center for Theoretical Physics, Baku AZ109641 Mehseti Str., Azerbaijan. [3]Department of Mathematics, University of Management and Technology, Lahore, Pakistan. [4]Department of Mathematics, College of Science, Al-Baha University, Alaqiq 65799, Saudi Arabia. [5]Department of Computer Science and Informatics, Applied College, Taibah University, Madinah 42353, Saudi Arabia. [6]Jadara University Research Center, Jadara University, IrbidPO Box 733, Jordan. [7]Applied Science Research Center, Amman, Jordan. [8]Department of Mathematics, Faculty of Science, Nangarhar 2601, Afghanistan. [9]Statistics and Operations Research Department, College of Science, Riyadh 11451, Kingdom of Saudi Arabia. ✉email: nazarmoh2001@gmail.com

Over the past several decades, extensive research has been conducted on nonlinear evolution equations (NLEEs)[6–9]. These equations have numerous applications across various areas of nonlinear science, including physics, mechanics, fiber optics, hydromechanics, and other disciplines that employ such models to describe complex physical processes[10–14]. Nonlinear equations are widely used in mathematical physics and engineering and have gained considerable importance, thereby driving the need for comprehensive understanding and analysis. Optical solitons, which are localized solitary wave packets, play a significant role in enabling high-speed data transmission through fiber optics and powering all-optical devices. The convergence of mathematical sciences, computational technology, and practical applications highlights the growing prominence of nonlinear partial differential equations in modern scientific research. One field that has recently attracted significant attention is the study of optical solitons in nonlinear materials.

The use of symbolic computations has also led to the development of numerous effective methods for obtaining soliton solutions in a wide range of nonlinear systems[15–19]. Examples of such models include the nonlinear Schrödinger equations[20–22], the fractional Phi–4 equation[23], the Nizhnik–Novikov–Veselov equation[24,25], the Manakov equation[26], the Akbota equation[27], the Whitham–Broer–Kaup equation[28], the Kadomtsev–Petviashvili modified equal width equation[29], and the integrable Kairat–X equation[30]. Nonlinear models are extensively applied in diverse fields such as neural networks, economics, natural sciences, data analysis, image processing, optics, plasma physics, and fluid dynamics. Optical solutions of NLPDEs have been derived through a variety of analytical techniques. These include the generalized exponential rational function approach[31], improved modified extended tanh function approach[32], the $(G^{'}/G)-$expansion approach[33], auxiliary equation approach[34], trial equation method[35], the planar dynamical system approach[36], extended simple equation approach[37–39], modified Sardar sub equation approach[40], sine–cosine method[41], improved F-expansion approach[42], exp $(-\Phi(\eta))-$function technique[43], Kudryshov auxiliary equation approach[44], the extended modified rational expansion approach[45,46], the Riccati Bernoulli sub optimal differential equation approach[47], extended direct algebraic mapping approach[48], and modified extended auxiliary equation mapping method[49]. Other techniques have also been employed to solve various nonlinear models, such as sensitivity analysis and finite-difference time-domain methods[50,51]. These approaches are among the most widely accepted methods for generating exact solutions of NLPDEs. Solitary wave solutions can be obtained from different nonlinear scenarios using relatively simple procedures. Moreover, these methods have the advantage of being applicable to problems involving large balancing numbers.

The integrable S-IIA equation is a nonlinear partial differential equation that plays a prominent role in quantum field theory and quantum mechanics. It describes the behavior of elementary particles as well as the propagation of dislocations in crystals. Research on the S-IIA equation has primarily focused on the recurrence nature of initial states, optical soliton interactions in collisionless fluids, solitons in condensed matter physics, and nonlinear wave equations.

In this study, the proposed complex nonlinear Shynaray-IIA system is given as[52–56]:

$$\begin{aligned} &iq_t + q_{xt} - i(vq)_x = 0, \\ &ir_t - r_{xt} - i(rv)_x = 0, \\ &v_x - \frac{\mu^2}{\varpi}(qr)_t = 0. \end{aligned} \tag{1}$$

Here, μ and ϖ are constants, while v, r, and q are the complex unknown functions that depend on the spatial variable x and time t. In the past, several studies have investigated the S-IIA equation using different approaches. Recently, Faridi et al. explored soliton solutions of the S-IIA equation by employing the Sardar sub equation and extended direct algebraic approaches[52]. Tipu et al. obtained soliton solutions of the S-IIA equation through the Φ^6-model expansion technique[53]. Altalbe et al. derived exact fractional soliton solutions of the M-fractional S-IIA equation using three schemes, namely the $\exp_a$ function method, the modified simple equation method, and the Kudryashov approach[54]. Khan et al. studied exact solutions of the S-IIA equation based on the improved Sardar sub equation approach[55]. Amer et al. determined soliton solutions of the nonlinear S-IIAE via the modified extended tanh scheme[56].

In this investigation, we examined various structural soliton and solitary wave solutions including peakon dark solitons, bell bright solitons, kink waves, peakon bright solitons, anti-kink waves, bell dark solitons, mixed bright and dark solitons, mixed kink bright and dark solitons, mixed anti-kink bright and dark solitons, and diverse forms of periodic wave solitons for the nonlinear complex S-IIA equation by employing the auxiliary equation approach. The main focus of this research is the examination of optical soliton solutions, which has broadened to encompass a variety of physical domains. The auxiliary equation method is one of the most widely accepted techniques for constructing soliton solutions of nonlinear partial differential equations[57,58]. Soliton solutions can be extracted from different nonlinear equations using this simple approach, which offers the added advantage of being applicable to problems involving large balancing numbers. The remaining sections of the present study are organized as follows: Section 2 provides a brief description of the proposed methodology, namely the auxiliary equation method. Section 3 presents the mathematical formulation of the proposed system. Section 4 applies the proposed approach to the governing model to examine the varieties of soliton solutions. Section 5 offers a graphical explanation of the obtained solutions. Section 6 discusses the novelty and innovation of the work. Finally, Section 7 presents the conclusion and outlines the study's perspective.

Brief description of proposed methodology

The nonlinear equation with partial derivative of space and time consider as:

$$R(q, q_t, q_x, q_{tt}, q_{xx}, ...) = 0. \tag{2}$$

While R called function of polynomial to the q_t, q_x. To get the nonlinear ordinary differential equation (NLODE), then apply the transform of wave as:

$$q(x,t) = \hbar(\zeta)\, e^{i\Theta}, \quad \Theta = -\kappa x + \Lambda t + \Xi, \qquad \zeta = x - \wp t. \tag{3}$$

Where κ, Λ, Ξ, and $\wp$ are parameters. Substituting Eq.(3) in Eq.(2), then obtained nonlinear ordinary differential equation (NLODE).

$$S(\hbar, \hbar_\zeta, \hbar_{\zeta\zeta}, \hbar_{\zeta\zeta\zeta}, ...) = 0. \tag{4}$$

Let us the solution in generalized form of Eq.(4) given as:

$$\hbar(\zeta) = \sum_{i=0}^{n} b_i \varphi^i(\zeta). \tag{5}$$

The function $\varphi(\zeta)$ satisfying the given equation.

$$\left(\varphi'(\zeta)\right)^2 = \nabla_1 \varphi^2(\zeta) + \nabla_2 \varphi^3(\zeta) + \nabla_3 \varphi^4(\zeta). \tag{6}$$

While $\nabla_1,\ \nabla_2,\ \nabla_3$, are parameters. The Eq.(6) having these exact solutions as: **Case-I**

$$\varphi(x) = -\frac{\nabla_1 \nabla_2 \operatorname{sech}^2\left(\frac{\sqrt{\nabla_1}\zeta}{2}\right)}{\nabla_2^2 - \nabla_1 \nabla_3 \left(1 - \tanh\left(\frac{\sqrt{\nabla_1}\zeta}{2}\right)\right)^2}, \quad \text{when} \quad \nabla_1 > 0.$$

Case-II

$$\varphi(x) = \frac{2\nabla_1 \operatorname{sech}\left(\sqrt{\nabla_1}\zeta\right)}{\sqrt{\nabla_2^2 - 4\nabla_1\nabla_3} - \operatorname{sech}\left(\sqrt{\nabla_1}\zeta\right) - \nabla_2}, \quad \text{when} \quad \nabla_2^2 - 4\nabla_1\nabla_3 > 0.$$

To calculate the value of integer n then apply the homogeneous balance rule on Eq.(4). Substitute the Eq.(5) in Eq.(4) and collect each cofactors of $\varphi^i(\zeta)$, then to secure the algebraic equations then make each cofactors equal to zero. To calculate the unknown values to solve the algebraic equations through any computation tool. Inserting the calculated values in Eq.(5) and examined the exact solutions of Eq.(2).

The complex nonlinear Shynaray-IIA equation

If $r = \epsilon\bar{q}$, $(\epsilon = \pm 1)$, then S-IIA equation secured in this form as:

$$\begin{aligned} &iq_t + q_{xt} - i(qv)_x = 0, \\ &v_x - \frac{\mu^2 \epsilon}{\varpi}\left(|q|^2\right)_t = 0. \end{aligned} \tag{7}$$

While ϖ, μ and ϵ are ordinary parameters. The wave transformation taken as

$$\begin{aligned} q(x,t) &= \hbar(\zeta)e^{i\Theta}, \qquad v(x,t) = \psi(\zeta), \\ \Theta &= -\kappa x + \Lambda t + \Xi, \qquad \zeta = x - \wp t. \end{aligned} \tag{8}$$

Where $\kappa, \Xi, \wp, \Lambda$ are called the ordinary constants. Set the Eq.(8) in the first part of Eq.(7). We separate imaginary and real parts secured as:

$$\wp\hbar'' + \Lambda(1-\kappa)\hbar + \kappa\hbar\psi + i\left(\Lambda - \wp(1-\kappa)\right)\hbar' - \hbar'\psi - \hbar\psi' = 0. \tag{9}$$

$$\psi' + \frac{2\wp\epsilon\mu^2}{\varpi}\hbar\hbar' = 0. \tag{10}$$

We integrate the Eq.(10), secured as:

$$\psi = -\frac{\wp\epsilon\mu^2}{\varpi}\hbar^2. \tag{11}$$

Set Eq.(11) in Eq.(9), we separated the real and imaginary parts then secured the real part as:

$$\wp\hbar'' + \Lambda(1-\kappa)\hbar - \frac{\kappa\wp\epsilon\mu^2}{\varpi}\hbar^3 = 0. \tag{12}$$

Imaginary part secured as:

$$\left(\Lambda - \wp(1-\kappa)\right)\hbar' + \frac{3\wp\epsilon\mu^2}{\varpi}\hbar''\hbar' = 0. \tag{13}$$

Construction of optical soliton solutions of governing model

Applied homogeneous balance rule on Eq.(12), examined $n = 1$. The generalized solutions given as:

$$\hbar(\zeta) = b_0 + b_1\varphi(\zeta). \tag{14}$$

Substitute the Eq.(9) in Eq.(8) and collect each cofactors of $\varphi^i(\zeta)$, then secured the algebraic equations by making each cofactors equal to zero. Solved these equations through Mathematica tool and calculated the unknown values as:

Set-I

$$b_0 = -\frac{\nabla_2\sqrt{\varpi}}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}, \quad b_1 = -\frac{\sqrt{2}\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\mu}, \quad \kappa = 1, \quad \nabla_1 = \frac{\nabla_2^2\left(\nabla_2 + 12\nabla_3\right)}{32\nabla_3^2}. \tag{15}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(15) into Eq.(14).

$$q_1(x,t) = \left(\frac{\nabla_2\sqrt{\varpi}\left(\nabla_2^2 - \nabla_1\nabla_3\left(\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 + 4\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)\right)\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu\left(\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 - \nabla_2^2\right)}\right) e^{i(-\kappa x + \Lambda t + \Xi)}, \tag{16}$$

$$v_1(x,t) = -\frac{\wp\epsilon\mu^2}{\varpi}\left(\frac{\nabla_2\sqrt{\varpi}\left(\nabla_2^2 - \nabla_1\nabla_3\left(\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 + 4\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)\right)\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu\left(\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 - \nabla_2^2\right)}\right)^2 e^{i(-\kappa x + \Lambda t + \Xi)}, \tag{17}$$

$$q_2(x,t) = \left(-\frac{\nabla_2\sqrt{\varpi}\left(\nabla_2^2 - \nabla_1\nabla_3\left(\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 + 4\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)\right)\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu\left(\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 - \nabla_2^2\right)}\right) e^{i(-\kappa x + \Lambda t + \Xi)}, \tag{18}$$

$$v_2(x,t) = -\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{\nabla_2\sqrt{\varpi}\left(\nabla_2^2 - \nabla_1\nabla_3\left(\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 + 4\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)\right)\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu\left(\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2 - \nabla_2^2\right)}\right)^2 e^{i(-\kappa x + \Lambda t + \Xi)}, \tag{19}$$

Set-II

$$b_0 = \frac{\nabla_2\sqrt{\varpi}}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}, \quad b_1 = \frac{\sqrt{2}\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\mu}, \quad \kappa = 1, \quad \nabla_1 = \frac{\nabla_2^2\left(\nabla_2 + 12\nabla_3\right)}{32\nabla_3^2}. \tag{20}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(20) into Eq.(14).

$$q_3(x,t) = \left(\frac{\sqrt{\varpi}\left(\frac{8\nabla_1\nabla_3\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2 - \sqrt{\nabla_2^2 - 4\nabla_1\nabla_3} + \text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)} - \nabla_2\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}\right) e^{i(-\kappa x + \Lambda t + \Xi)}, \tag{21}$$

$$v_3(x,t) = -\frac{\wp\epsilon\mu^2}{\varpi}\left(\frac{\sqrt{\varpi}\left(\frac{8\nabla_1\nabla_3\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2 - \sqrt{\nabla_2^2 - 4\nabla_1\nabla_3} + \text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)} - \nabla_2\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}\right)^2 e^{i(-\kappa x + \Lambda t + \Xi)}. \tag{22}$$

$$q_4(x,t)=\left(\frac{\sqrt{\varpi}\left(\nabla_2-\frac{8\nabla_1\nabla_3\operatorname{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\operatorname{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{23}$$

$$v_4(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(\frac{\sqrt{\varpi}\left(\nabla_2-\frac{8\nabla_1\nabla_3\operatorname{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\operatorname{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)}{2\sqrt{2}\sqrt{\epsilon}\sqrt{\nabla_3}\mu}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{24}$$

Set-III

$$b_0=b_0,\quad b_1=-\frac{\sqrt{2}\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\mu},\quad \kappa=1,\quad \nabla_1=\frac{3b_0^2\epsilon\mu^2}{\varpi}-\frac{b_0^3\epsilon^{3/2}\mu^3}{\sqrt{2}\sqrt{\nabla_3}\varpi^{3/2}},\quad \nabla_2=-\frac{2\sqrt{2}b_0\sqrt{\epsilon}\sqrt{\nabla_3}\mu}{\sqrt{\varpi}}. \tag{25}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(25) into Eq.(14).

$$q_5(x,t)=\left(b_0+\frac{\sqrt{2}\nabla_1\nabla_2\sqrt{\nabla_3}\sqrt{\varpi}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\sqrt{\epsilon}\mu\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{26}$$

$$v_5(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0+\frac{\sqrt{2}\nabla_1\nabla_2\sqrt{\nabla_3}\sqrt{\varpi}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\sqrt{\epsilon}\mu\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{27}$$

$$q_6(x,t)=\left(b_0+\frac{2\sqrt{2}\nabla_1\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\left(\mu+\nabla_2\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right) e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{28}$$

$$v_6(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0+\frac{2\sqrt{2}\nabla_1\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\left(\mu+\nabla_2\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{29}$$

Set-IV

$$b_0=b_0,\quad b_1=\frac{\sqrt{2}\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\mu},\quad \kappa=1,\quad \nabla_1=\frac{b_0^3\epsilon^{3/2}\mu^3}{\sqrt{2}\sqrt{\nabla_3}\varpi^{3/2}}+\frac{3b_0^2\epsilon\mu^2}{\varpi},\quad \nabla_2=\frac{2\sqrt{2}b_0\sqrt{\epsilon}\sqrt{\nabla_3}\mu}{\sqrt{\varpi}}. \tag{30}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(30) into Eq.(14).

$$q_7(x,t)=\left(b_0-\frac{\sqrt{2}\nabla_1\nabla_2\sqrt{\nabla_3}\sqrt{\varpi}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\sqrt{\epsilon}\mu\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{31}$$

$$v_7(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0-\frac{\sqrt{2}\nabla_1\nabla_2\sqrt{\nabla_3}\sqrt{\varpi}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\sqrt{\epsilon}\mu\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{32}$$

$$q_8(x,t)=\left(b_0+\frac{2\sqrt{2}\nabla_1\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\left(\mu+\nabla_2\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right) e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{33}$$

$$v_8(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0+\frac{2\sqrt{2}\nabla_1\sqrt{\nabla_3}\sqrt{\varpi}}{\sqrt{\epsilon}\left(\mu+\nabla_2\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\mu\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{34}$$

Set-V

$$b_0=0,\quad b_1=b_1,\quad \kappa=\frac{2\nabla_3\varpi}{b_1^2\epsilon\mu^2},\quad \nabla_1=\frac{\Lambda\left(\frac{2\nabla_3\varpi}{b_1^2\epsilon\mu^2}-1\right)}{\wp}. \tag{35}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(35) into Eq.(14).

$$q_9(x,t)=\left(\frac{b_1\nabla_1\nabla_2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2-\nabla_2^2}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{36}$$

$$v_9(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(\frac{b_1\nabla_1\nabla_2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2-\nabla_2^2}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{37}$$

$$q_{10}(x,t)=\left(-\frac{2b_1\nabla_1\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{38}$$

$$v_{10}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{2b_1\nabla_1\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{39}$$

Set-VI

$$b_0=b_0,\quad b_1=b_1,\quad \kappa=1,\quad \varpi=\frac{b_1^2\epsilon\mu^2}{2\nabla_3},\quad \nabla_1=\frac{2b_0^2\left(b_0+3b_1\right)\nabla_3}{b_1^3},\quad \nabla_2=\frac{4b_0\nabla_3}{b_1}. \tag{40}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(40) into Eq.(14).

$$q_{11}(x,t)=\left(\frac{b_1\nabla_1\nabla_2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2-\nabla_2^2}+b_0\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{41}$$

$$v_{11}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(\frac{b_1\nabla_1\nabla_2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2-\nabla_2^2}+b_0\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{42}$$

$$q_{12}(x,t)=\left(b_0-\frac{2b_1\nabla_1\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{43}$$

$$v_{12}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0-\frac{2b_1\nabla_1\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{44}$$

Set-VII

$$b_0=b_0,\quad b_1=\frac{b_0^3\epsilon\mu^2}{\nabla_1\varpi-3b_0^2\epsilon\mu^2},\quad \kappa=1,\quad \nabla_2=\frac{2b_0^4\epsilon^2\mu^4}{\varpi\left(\nabla_1\varpi-3b_0^2\epsilon\mu^2\right)},\quad \nabla_3=\frac{b_0^6\epsilon^3\mu^6}{2\varpi\left(\nabla_1\varpi-3b_0^2\epsilon\mu^2\right)^2}. \tag{45}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(45) into Eq.(14).

$$q_{13}(x,t)=\left(b_0-\frac{b_0^3\epsilon\nabla_1\nabla_2\mu^2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\left(\nabla_1\varpi-3b_0^2\epsilon\mu^2\right)\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{46}$$

$$v_{13}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0-\frac{b_0^3\epsilon\nabla_1\nabla_2\mu^2\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{(\nabla_1\varpi-3b_0^2\epsilon\mu^2)\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{47}$$

$$q_{14}(x,t)=\left(b_0-\frac{2b_0^3\epsilon\nabla_1\mu^2\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{(\nabla_1\varpi-3b_0^2\epsilon\mu^2)\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{48}$$

$$v_{14}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0-\frac{2b_0^3\epsilon\nabla_1\mu^2\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{(\nabla_1\varpi-3b_0^2\epsilon\mu^2)\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{49}$$

Set-VIII

$$b_0=\frac{1}{2}\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}},\quad b_1=\frac{\nabla_2\varpi}{\epsilon\mu^2\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}},$$
$$\kappa=1,\quad \nabla_3=\frac{1}{-\frac{2\sqrt{8\nabla_1+9\nabla_2}}{\nabla_2^{3/2}}-\frac{6}{\nabla_2}}. \tag{50}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(50) into Eq.(14).

$$q_{15}(x,t)=\left(-\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}+\frac{1}{2}\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{51}$$

$$v_{15}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}+\frac{1}{2}\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{52}$$

$$q_{16}(x,t)=\left(-\frac{2\nabla_1\nabla_2\varpi\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}+\frac{1}{2}\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{53}$$

$$v_{16}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{2\nabla_1\nabla_2\varpi\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}+\frac{1}{2}\sqrt{-\frac{3\nabla_2\varpi+\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi}{\epsilon\mu^2}}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{54}$$

Set-IX

$$b_0=b_0,\quad b_1=\frac{\nabla_2\varpi}{2b_0\epsilon\mu^2},\quad \kappa=1,\quad \nabla_3=\frac{\nabla_2^2\varpi}{8b_0^2\epsilon\mu^2}$$
$$\nabla_1=\frac{b_0^2\epsilon\mu^2\left(2b_0^2\epsilon\mu^2+3\nabla_2\varpi\right)}{\nabla_2\varpi^2}. \tag{55}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(55) into Eq.(14).

$$q_{17}(x,y)=\left(b_0-\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{2b_0\epsilon\mu^2\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{56}$$

$$v_{17}(x,y)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(b_0-\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{2b_0\epsilon\mu^2\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{57}$$

$$q_{18}(x,t)=\left(\frac{\nabla_1\nabla_2\varpi}{b_0\epsilon\mu^2\left(-\nabla_2\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)+\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-1\right)}+b_0\right) e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{58}$$

$$v_{18}(x,t)=\left(\frac{\nabla_1\nabla_2\varpi}{b_0\epsilon\mu^2\left(-\nabla_2\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)+\sqrt{\nabla_2^2-4\nabla_1\nabla_3}\cosh\left(\sqrt{\nabla_1}(x-\wp t)\right)-1\right)}+b_0\right)^2 \left(-\frac{\wp\epsilon\mu^2}{\varpi}\right)e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{59}$$

Set-X

$$b_0=-\frac{1}{2}\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}},\quad b_1=-\frac{\nabla_2\varpi}{\epsilon\mu^2\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}},$$
$$\kappa=1,\quad \nabla_3=\frac{1}{\frac{2\sqrt{8\nabla_1+9\nabla_2}}{\nabla_2^{3/2}}-\frac{6}{\nabla_2}}. \tag{60}$$

The optical soliton solutions are examined in complex functions form of Eq.(1) through inserting the Eq.(60) into Eq.(14).

$$q_{19}(x,t)=\left(-\frac{1}{2}\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}+\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{61}$$

$$v_{19}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{1}{2}\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}+\frac{\nabla_1\nabla_2^2\varpi\text{sech}^2\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}\left(\nabla_2^2-\nabla_1\nabla_3\left(\tanh\left(\frac{1}{2}\sqrt{\nabla_1}(x-\wp t)\right)-1\right)^2\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{62}$$

$$q_{20}(x,t)=\left(-\frac{1}{2}\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}+\frac{2\nabla_1\nabla_2\varpi\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)e^{i(-\kappa x+\Lambda t+\Xi)}, \tag{63}$$

$$v_{20}(x,t)=-\frac{\wp\epsilon\mu^2}{\varpi}\left(-\frac{1}{2}\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}+\frac{2\nabla_1\nabla_2\varpi\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)}{\epsilon\mu^2\sqrt{\frac{\sqrt{\nabla_2}\sqrt{8\nabla_1+9\nabla_2}\varpi-3\nabla_2\varpi}{\epsilon\mu^2}}\left(\nabla_2-\sqrt{\nabla_2^2-4\nabla_1\nabla_3}+\text{sech}\left(\sqrt{\nabla_1}(x-\wp t)\right)\right)}\right)^2 e^{i(-\kappa x+\Lambda t+\Xi)}. \tag{64}$$

Graphical representation of the examined solutions

Graphical representation forms a key part of this research, as it provides insights into the physical structures of the examined solutions. We illustrated these structures using contour plots, as well as two- and three-dimensional plots, by assigning specific parameter values in the functions through the Mathematica tool. Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 display a variety of physical structures of solitons and solitary waves, including peakon bright, peakon dark, kink waves, periodic waves, bell bright, bell dark, anti-kink waves, mixed dark and bright solitons, mixed kink-bright waves, mixed kink-dark waves, and mixed anti-kink dark and bright waves. The graphical analysis highlights the influence of governing model parameters on the obtained solutions, with a particular focus on exploring their physical effects. Importantly, the figures serve to demonstrate the structural formation of solutions to the nonlinear model, with the emphasis placed on representing the dynamic and geometric features of the proposed model rather than validating accuracy for a specific real-world or experimental scenario. The detailed explanations of Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 are presented as follows:

Figure 1, visualized to peakon dark and periodic wave solitons for $q_1(x,t)$ function by utilizing these values $\nabla_0 = 2, \nabla_1 = 4, \nabla_2 = 2, \wp = 1.5, \kappa = 1, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$. Figure 2, visualized to peakon dark and periodic wave soliton structure of $v_1(x,t)$ function by utilizing these values $\nabla_0 = 1, \nabla_1 = 4, \nabla_2 = 1, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$. While Fig. 3, visualized to periodic wave soliton structure of $q_2(x,t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 3, \nabla_2 = 1, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$. Figure 4, visualized to periodic wave soliton structure of $v_2(x,t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 5, \nabla_2 = 3, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$. Figure 5, visualized to periodic wave soliton structure of $v_3(x,t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 3, \nabla_2 = 1, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$. Figure 6, visualized to periodic wave soliton structure in three different kinds of plotting of $q_4(x,t)$ function by utilizing these values

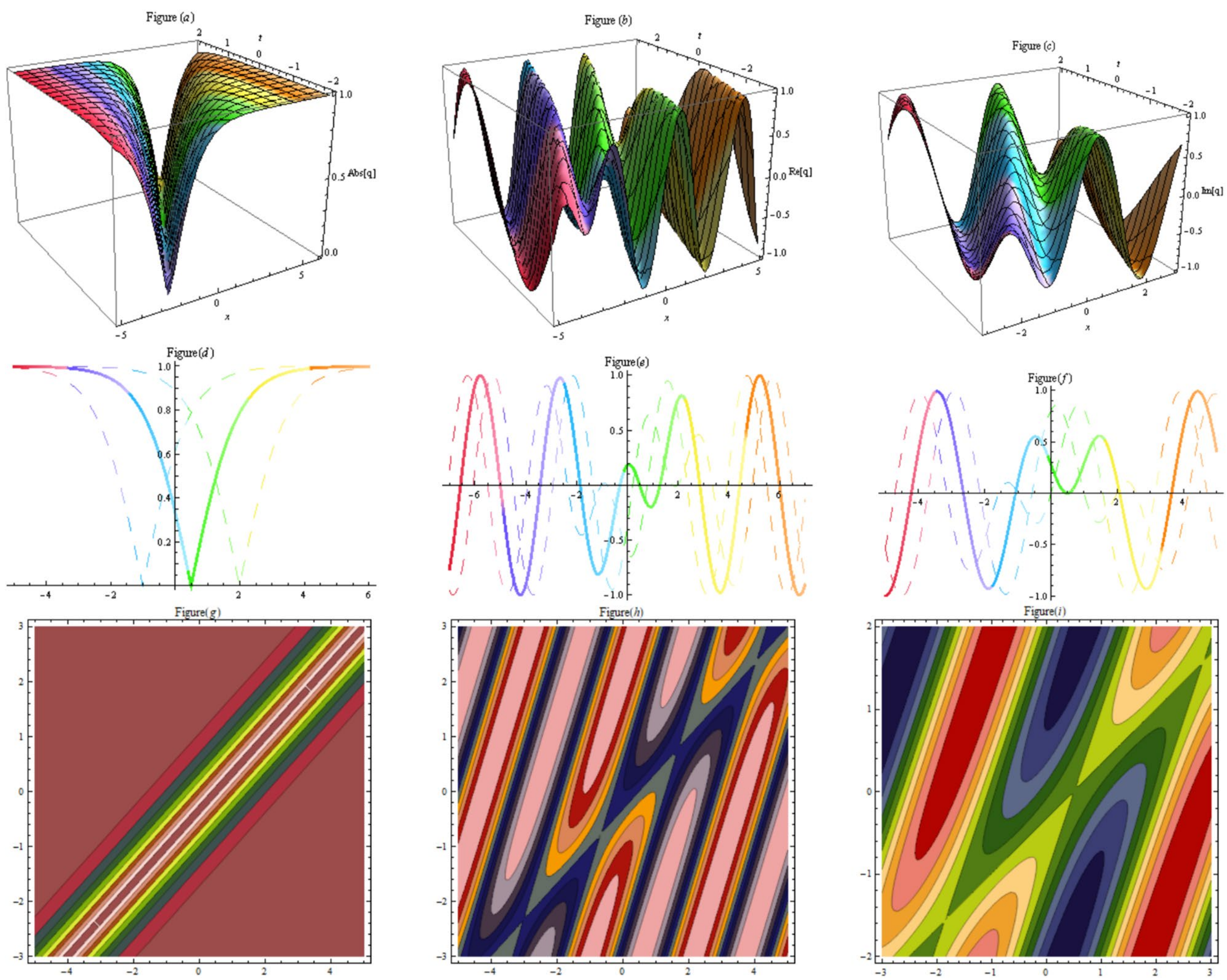

Fig. 1. Visualization of peakon dark and periodic wave soliton in three different kinds of plotting of $q_1(x,t)$ function by utilizing these values $\nabla_0 = 2, \nabla_1 = 4, \nabla_2 = 2, \wp = 1.5, \kappa = 1, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$.

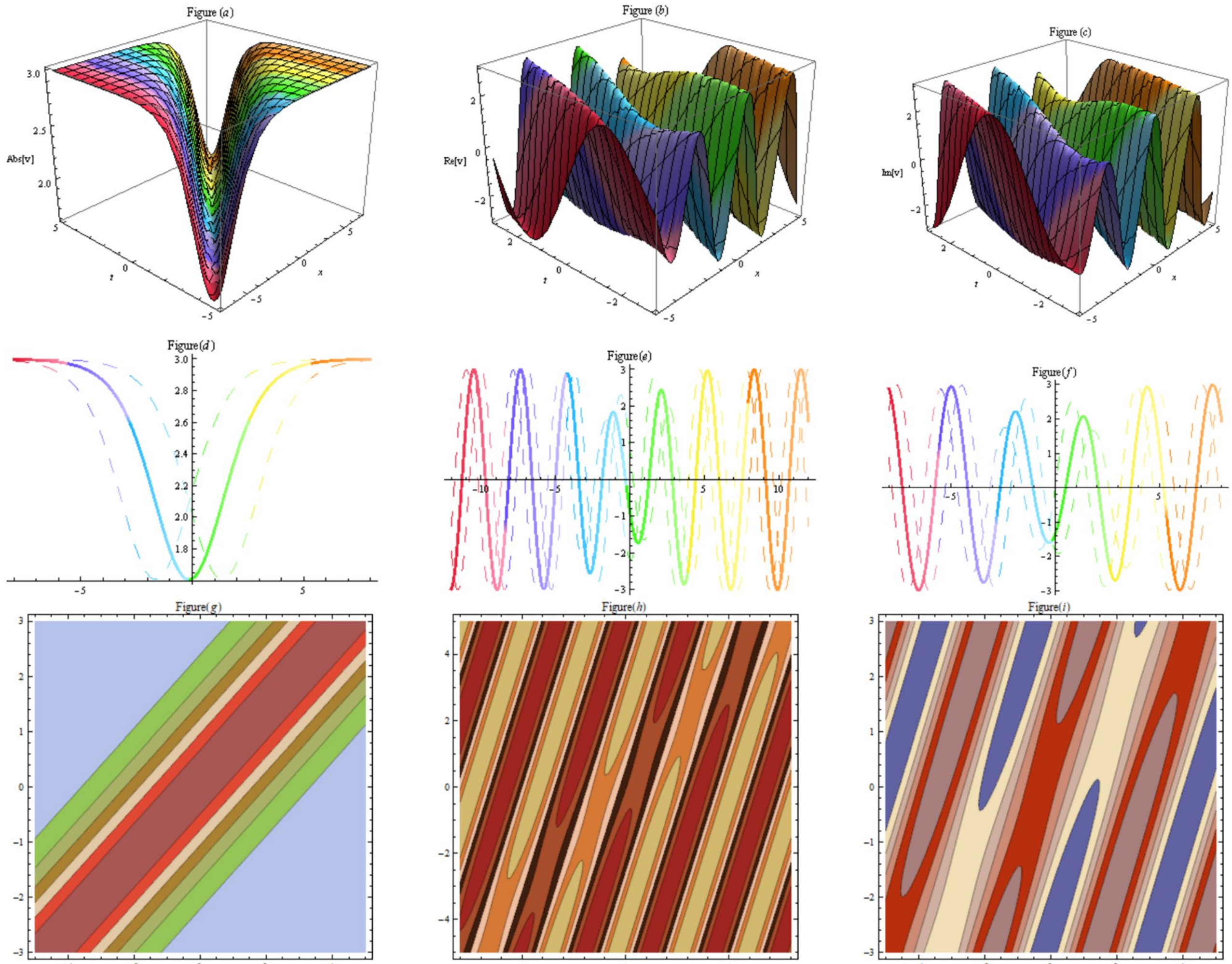

Fig. 2. Visualization of peakon dark and periodic wave soliton structure in three different kinds of plotting of $v_1(x,t)$ function by utilizing these values $\nabla_0=1, \nabla_1=4, \nabla_2=1, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1$.

$\nabla_0=-1, \nabla_1=5, \nabla_2=3, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1$. Figure 7, visualized to periodic wave soliton structure in three different kinds of plotting of $v_4(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-5, \nabla_2=1, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1$. Figure 8, visualized to peakon bright and periodic wave soliton structure in three different kinds of plotting of $q_5(x,t)$ function by utilizing these values $\nabla_0=2, \nabla_1=5, \nabla_2=2, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$. Figure 9, visualized to anti-kink and periodic wave soliton structure in three different kinds of plotting of $v_5(x,t)$ function by utilizing these values $\nabla_0=2, \nabla_1=4, \nabla_2=2, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=-2$. Figure 10, visualized to periodic wave soliton structure in three different kinds of plotting of $q_6(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-5, \nabla_2=1, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$. Figure 11, visualized to periodic wave soliton structure in three different kinds of plotting of $v_6(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-4, \nabla_2=3, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$. Figure 12, visualized to kink and periodic wave soliton structure in three different kinds of plotting of $q_7(x,t)$ function by utilizing these values $\nabla_0=2, \nabla_1=4, \nabla_2=2, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=-3$. And Fig. 13, visualized to periodic wave soliton structure in three different kinds of plotting of $q_8(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=4, \nabla_2=3, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$. The remaining solutions represent various nonlinear wave phenomena, including peakon bright and peakon dark solitons, kink and anti-kink waves, kink bright and kink dark waves, singular waves, mixed periodic waves, and singular periodic waves. The graphical representations of the explored solutions demonstrate the robustness and effectiveness of the proposed methodology. Moreover, the proposed approach shows strong capability in addressing complex problems by utilizing advanced symbolic computation tools.

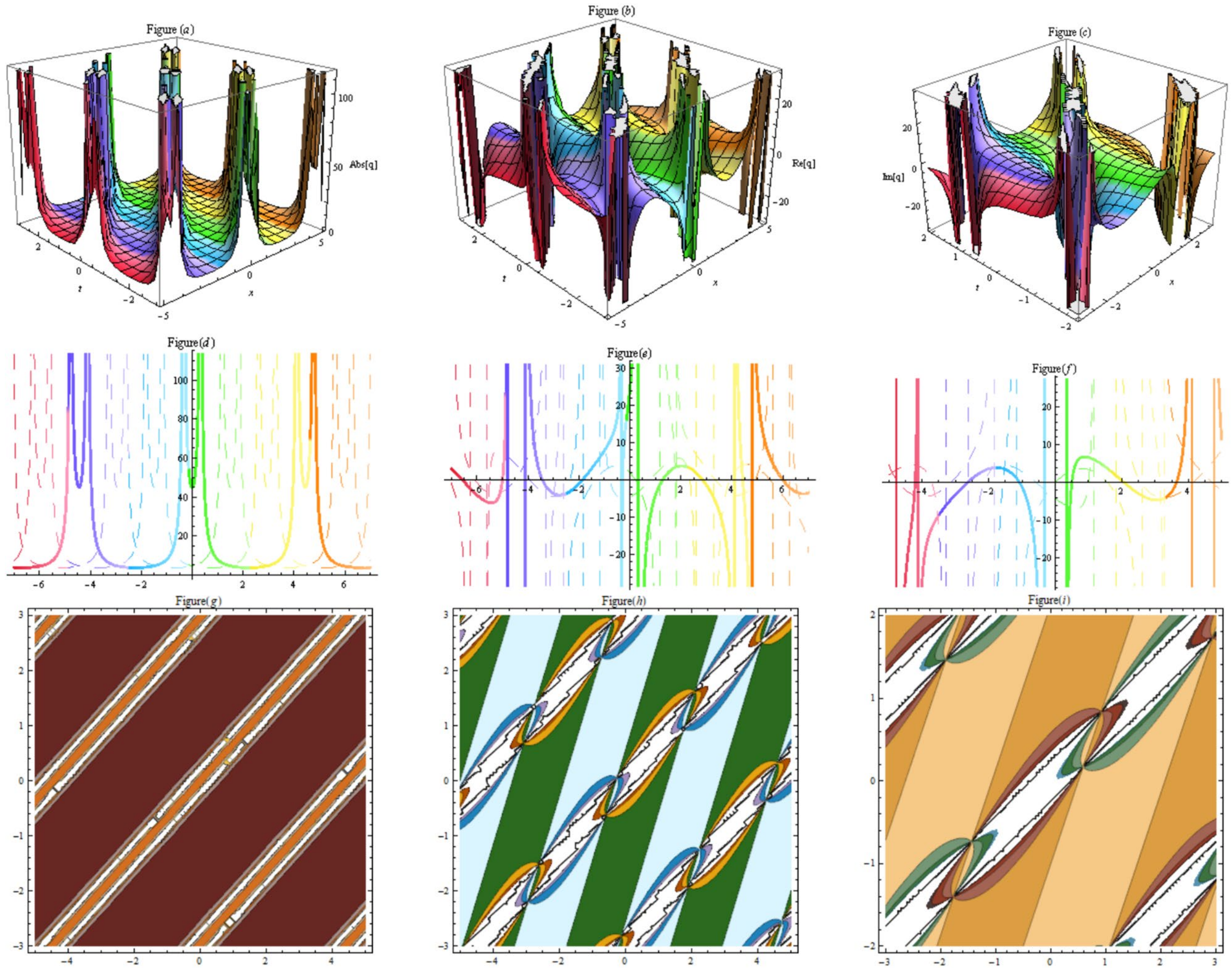

Fig. 3. Visualization of periodic wave soliton structure in three different kinds of plotting of $q_2(x, t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 3, \nabla_2 = 1, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1.$

Discussion of the results

In this section, we present novel and interesting results in the form of optical soliton and solitary wave solutions. The generated solutions are more generalized and innovative, obtained through an efficient integrated method. We also highlight the similarities and differences between our results for the nonlinear S-IIA equation and those previously reported in the literature.

Previously, several mathematicians and researchers investigated the integrable S-IIA equation using various methods, such as the Sardar sub-equation approach, extended direct algebraic approach, Φ^6-model expansion technique, $\exp_a$ function approach, modified simple equation approach, Kudryashov approach, improved Sardar sub-equation approach, inverse scattering transform approach, and modified extended tanh scheme[52–56]. The present study, however, explores different solutions beyond those existing approaches.

In contrast, our study examines the optical soliton wave patterns of the nonlinear S-IIA equation using a new derivative for the first time. We employ an analytical technique, namely the auxiliary equation method, which provides deeper insight into the behavior of optical solitons and wave propagation in fields such as quantum mechanics, optical communication, nonlinear optics, plasma physics, and fiber optics. This method enables the exploration of novel solutions expressed in trigonometric and hyperbolic functions. Using the auxiliary equation method, we obtained various solitons and solitary waves, as illustrated in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13, exhibiting different physical structures such as peakon bright, peakon dark, kink waves, periodic waves, bell bright, bell dark, anti-kink waves, mixed dark and bright solitons, mixed kink-bright waves, mixed kink-dark waves, and mixed anti-kink dark and bright waves.

Overall, the detailed comparison indicates that our methodology is more effective for finding exact solutions of higher-order NLPDEs in both complex and real forms. It is also easier to implement, more reliable, and highly efficient compared to previous methods.

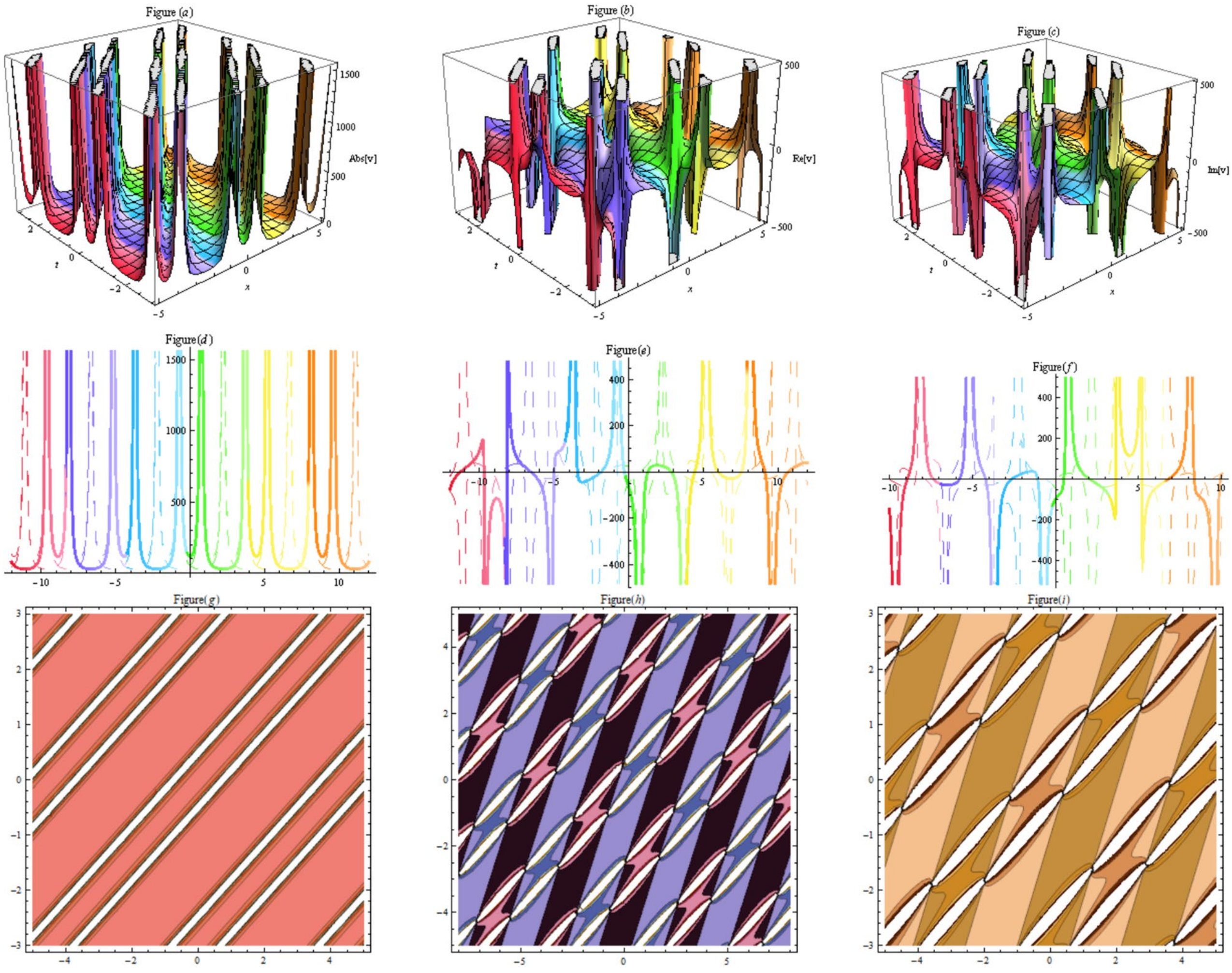

Fig. 4. Visualization of periodic wave soliton structure in three different kinds of plotting of $v_2(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=5, \nabla_2=3, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1$.

Conclusion

This research has thoroughly analyzed the behavior of optical soliton solutions of the nonlinear, integrable Shynaray-IIA equation. Various types of soliton solutions were obtained using the auxiliary equation method, including peakon bright and dark solitons, kink waves, periodic waves, bell bright and dark solitons, anti-kink waves, mixed dark and bright solitons, mixed kink-bright waves, mixed kink-dark waves, and mixed anti-kink dark and bright waves. The originality and diversity of these soliton solutions highlight the flexibility and efficiency of the proposed method and have significant implications for both theoretical understanding and practical applications in nonlinear optical systems.

This study establishes a solid foundation for future research and opens new prospects for developments in optical communication and related engineering fields. The results may stimulate further investigation into the interaction of optical soliton solutions with novel materials, such as photonic crystals and metamaterials. The exact soliton solutions obtained are valuable for understanding wave propagation and can serve to validate experimental and numerical results across various areas, including optics, quantum physics, fiber optics, quantum mechanics, plasma physics, nonlinear optics, biomedical engineering, and mathematical physics. Furthermore, Mathematica was used to generate three-dimensional, two-dimensional, and contour plots that illustrate the physical behavior of these solutions. The visualized physical structures demonstrate the robustness and effectiveness of the method. Importantly, this approach can also be applied to other nonlinear partial differential equations, making it a versatile tool for further studies in nonlinear science.

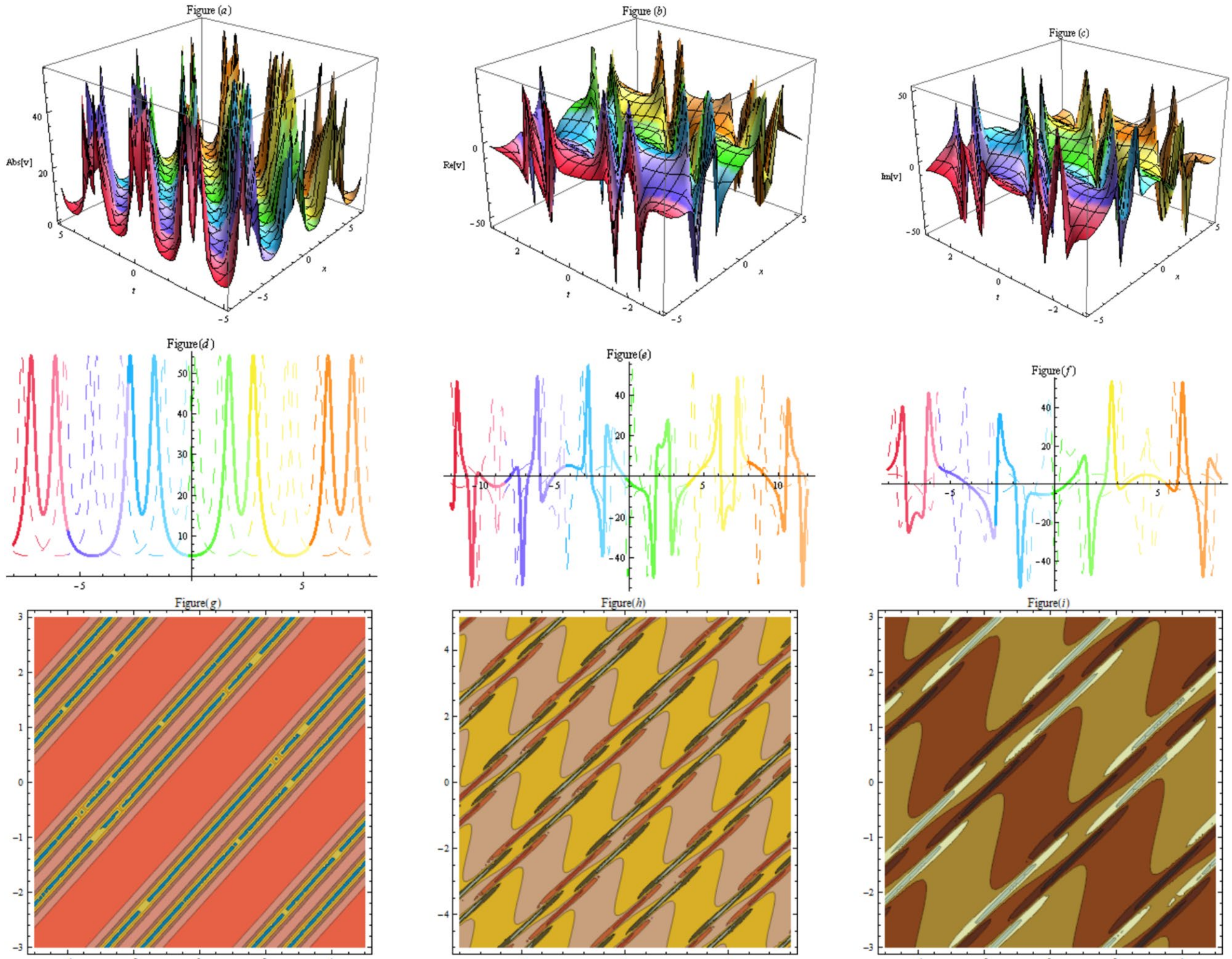

Fig. 5. Visualization of periodic wave soliton structure in three different kinds of plotting of $v_3(x,t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 3, \nabla_2 = 1, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$.

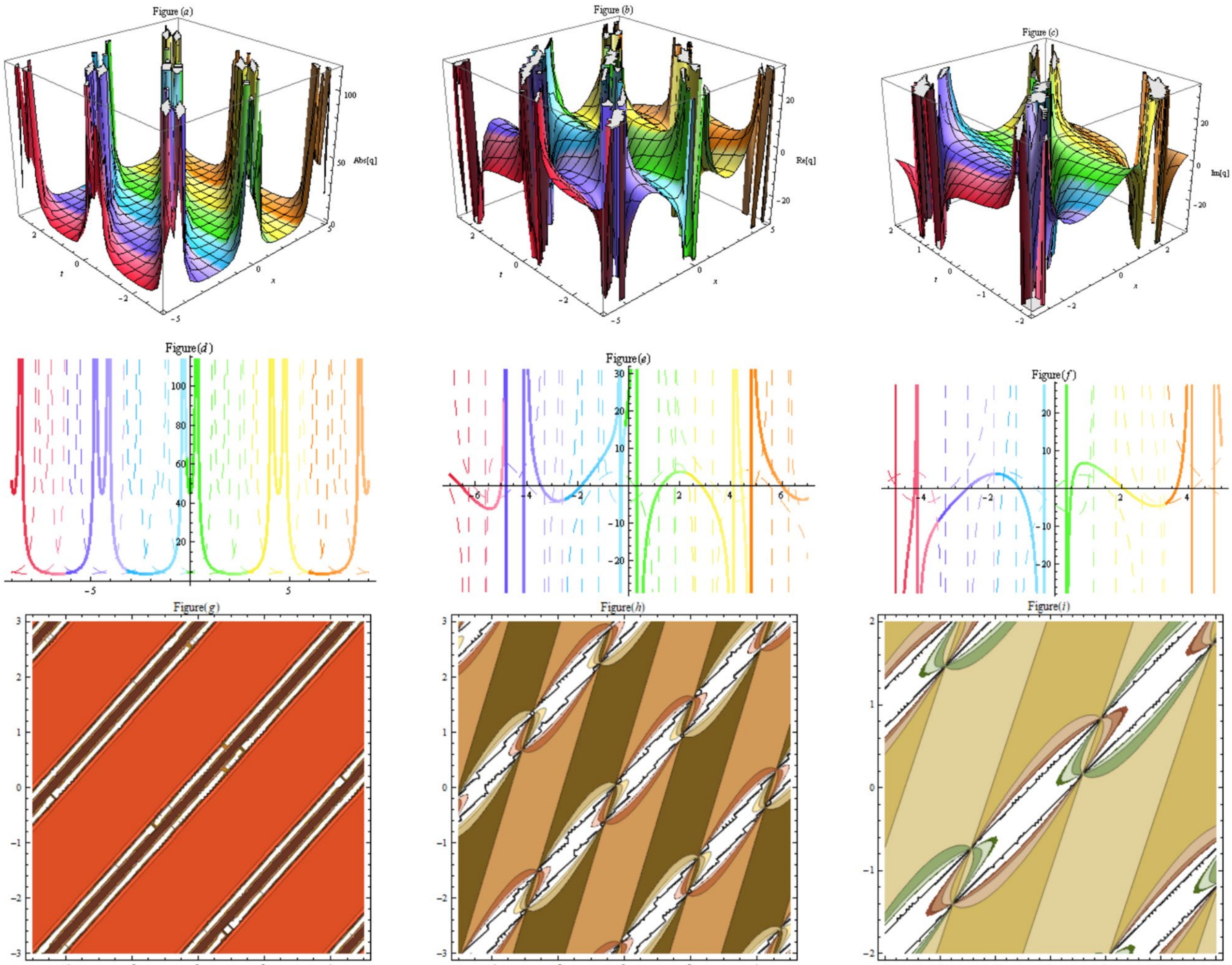

Fig. 6. Visualization of periodic wave soliton structure in three different kinds of plotting of $q_4(x,t)$ function by utilizing these values $\nabla_0 = -1, \nabla_1 = 5, \nabla_2 = 3, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1$.

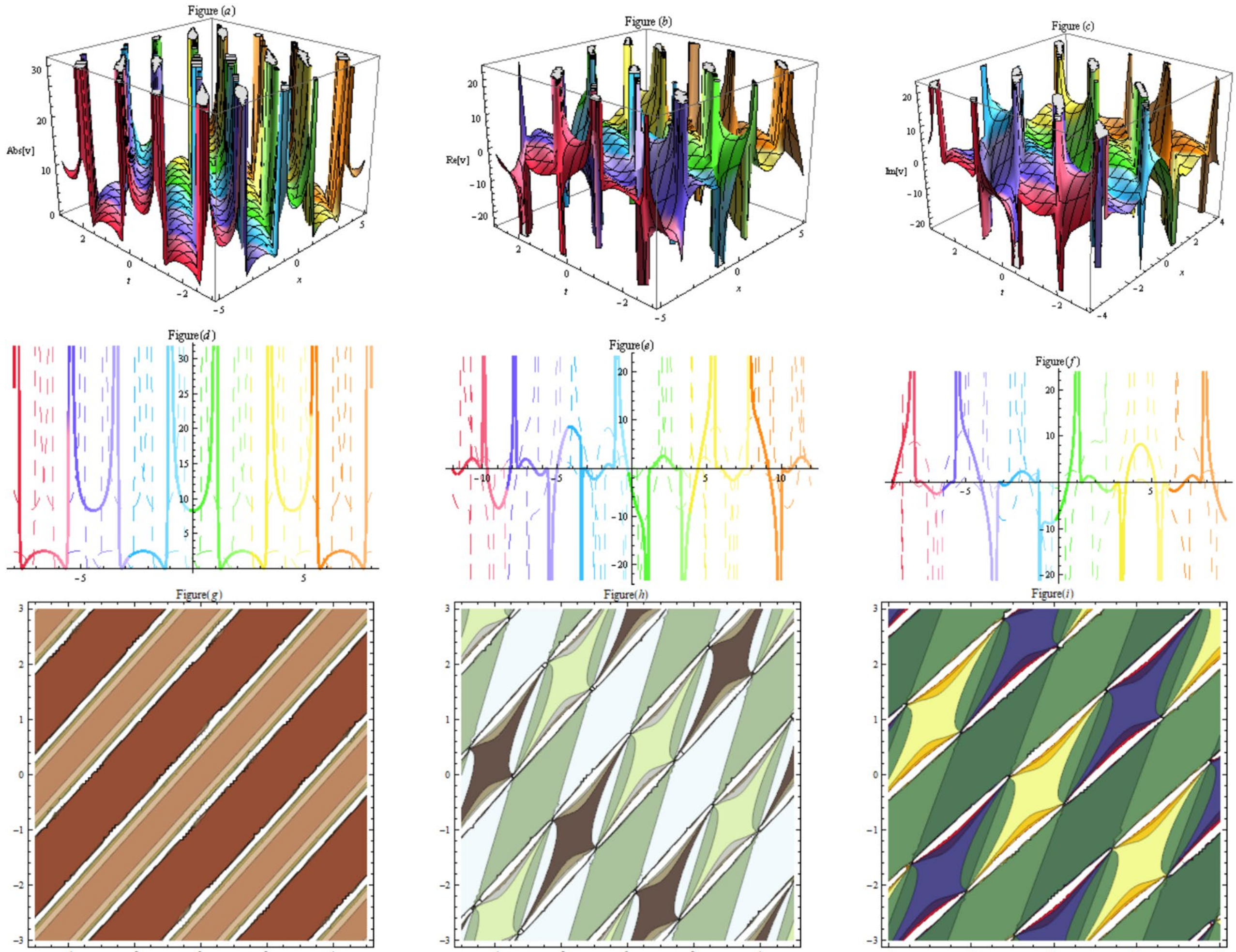

Fig. 7. Visualization of periodic wave soliton structure in three different kinds of plotting of $v_4(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-5, \nabla_2=1, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1$.

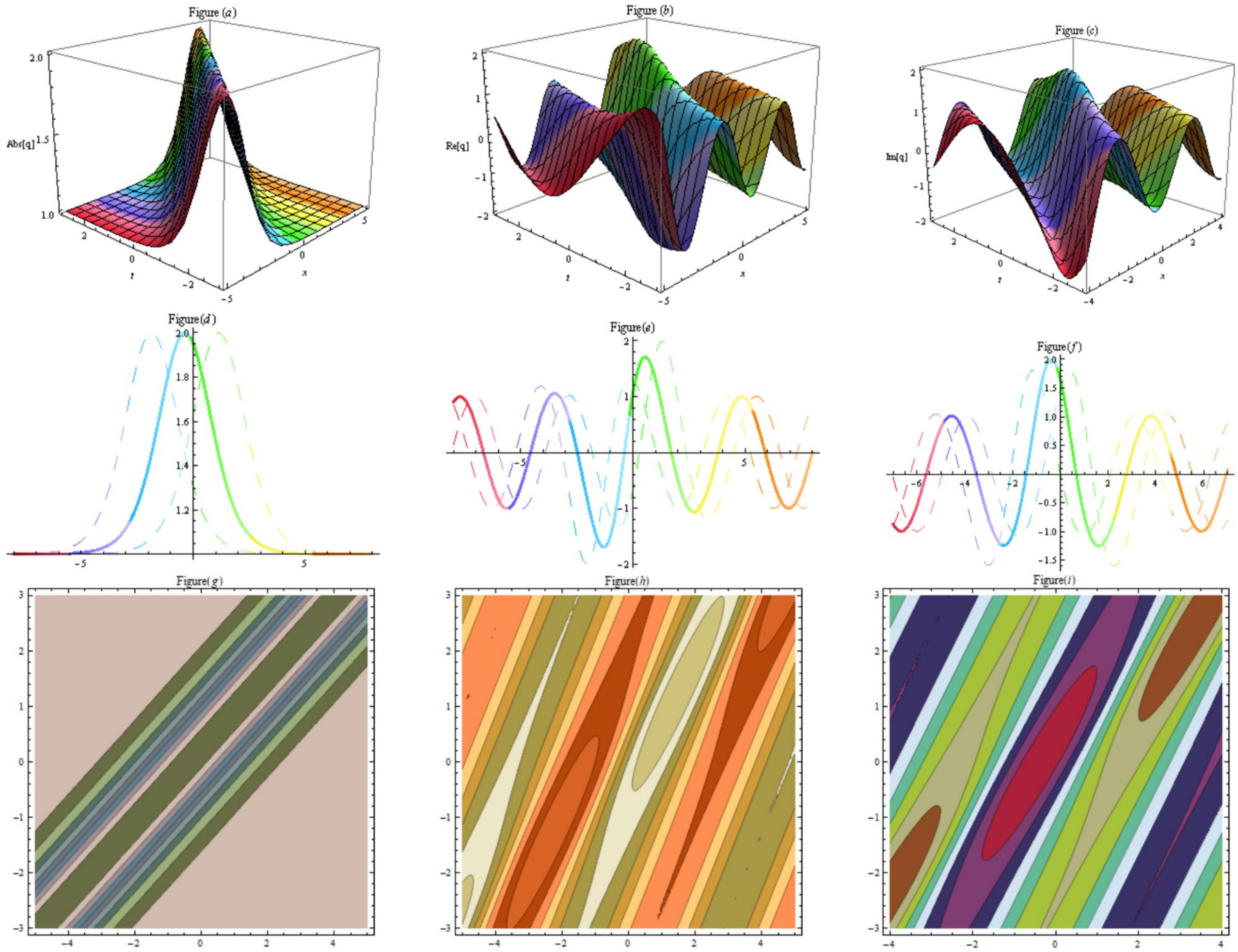

Fig. 8. Visualization of peakon bright and periodic wave soliton structure in three different kinds of plotting of $q_5(x,t)$ function by utilizing these values $\nabla_0=2, \nabla_1=5, \nabla_2=2, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$.

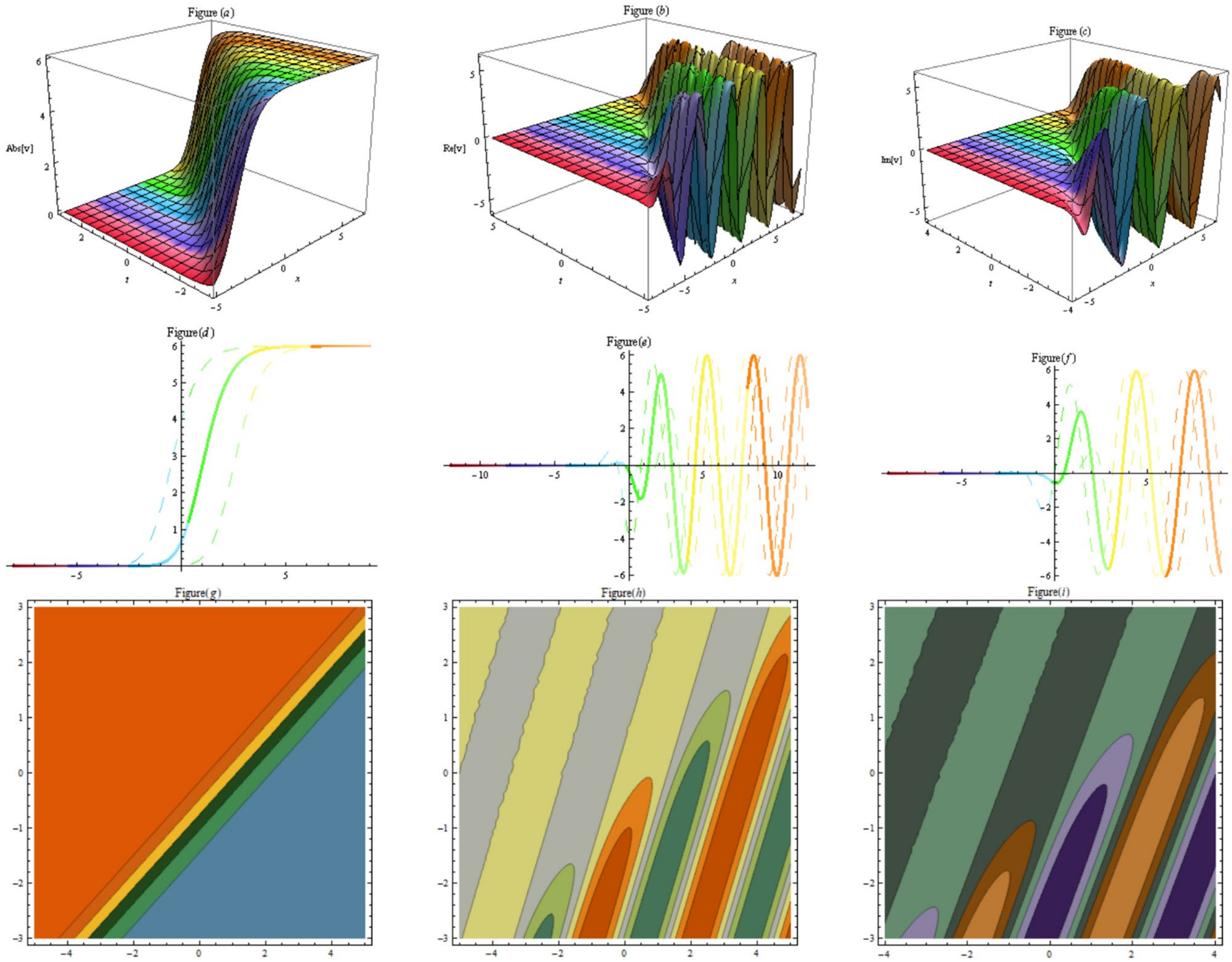

Fig. 9. Visualization of anti-kink and periodic wave soliton structure in three different kinds of plotting of $v_5(x,t)$ function by utilizing these values $\nabla_0 = 2, \nabla_1 = 4, \nabla_2 = 2, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1, b_0 = -2.$

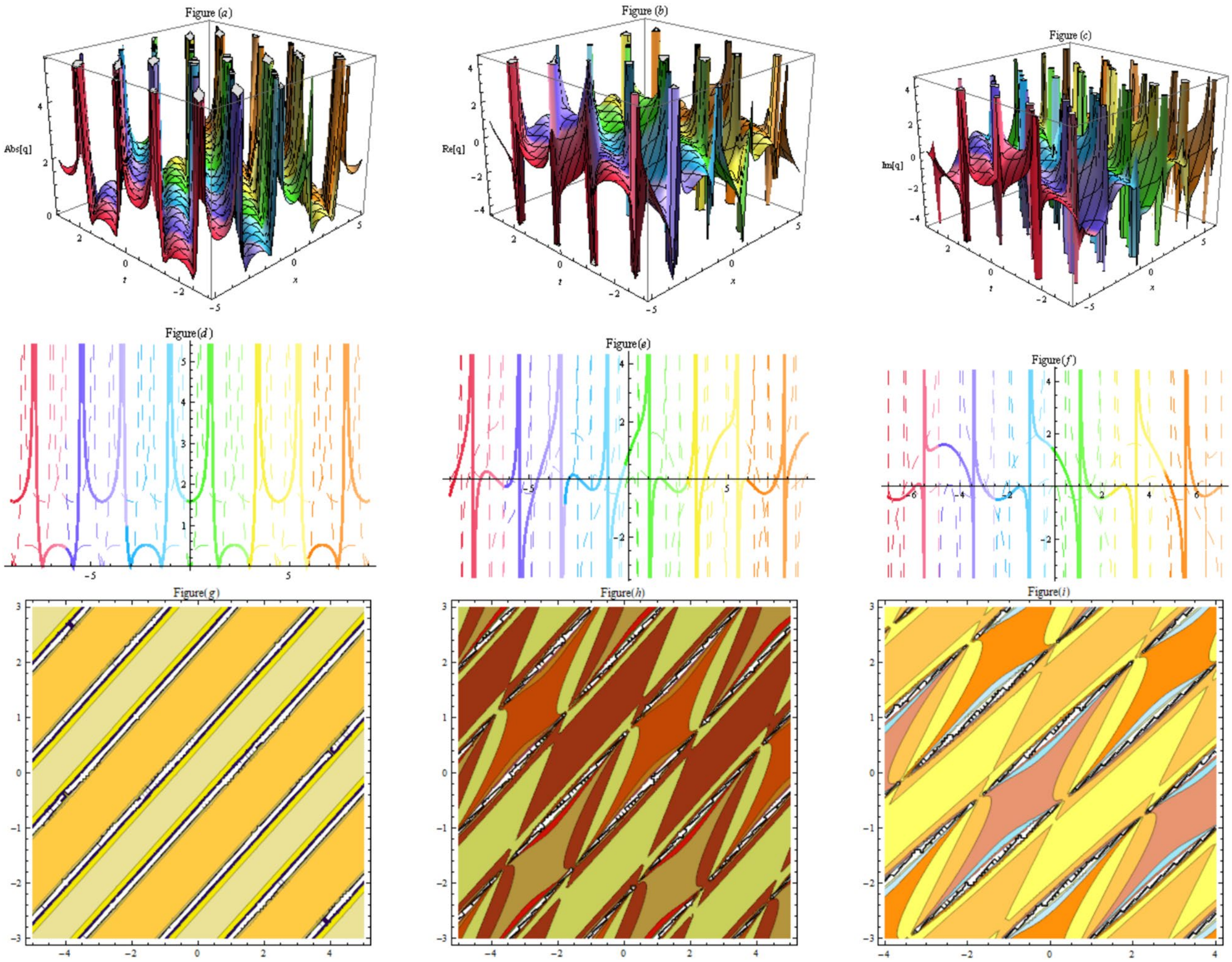

Fig. 10. Visualization of periodic wave soliton structure in three different kinds of plotting of $q_6(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-5, \nabla_2=1, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1.$

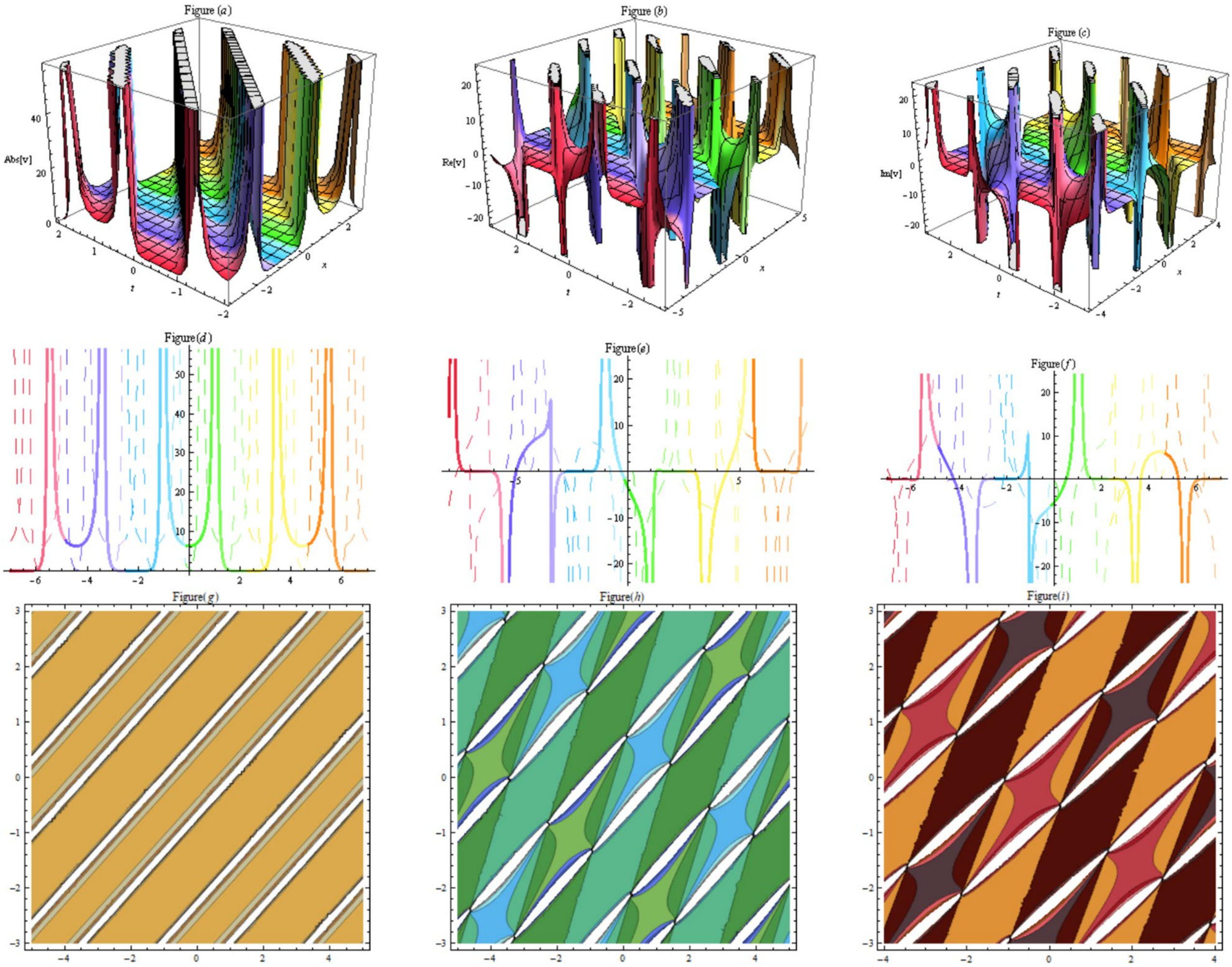

Fig. 11. Visualization of periodic wave soliton structure in three different kinds of plotting of $v_6(x,t)$ function by utilizing these values $\nabla_0=-2, \nabla_1=-4, \nabla_2=3, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=1$.

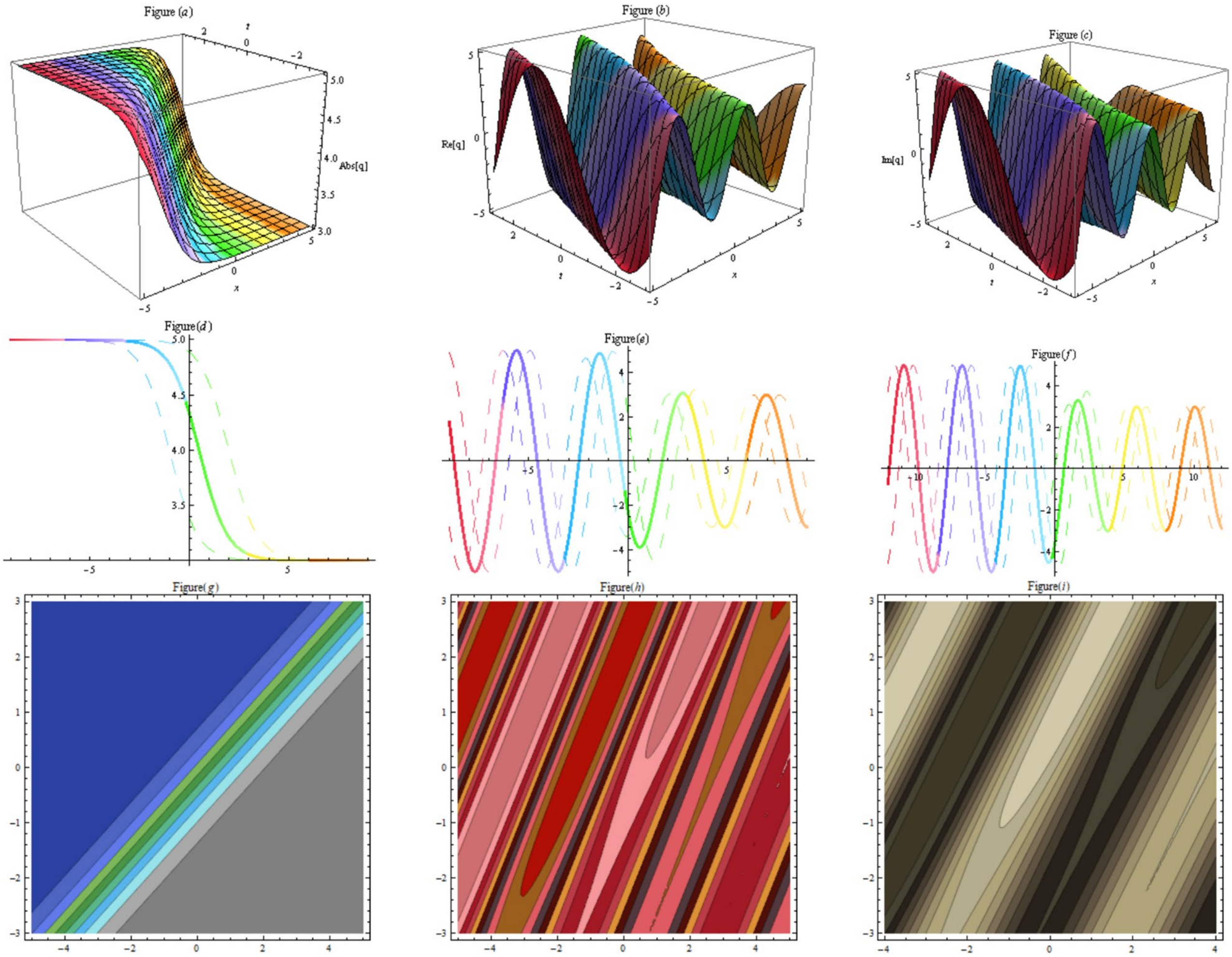

Figure 12. Visualization of kink and periodic wave soliton structure in three different kinds of plotting of $q_7(x,t)$ function by utilizing these values $\nabla_0=2, \nabla_1=4, \nabla_2=2, \kappa=1, \wp=1.5, \mu=1, \Lambda=1, \epsilon=1, \Xi=1, \varpi=1, b_0=-3.$

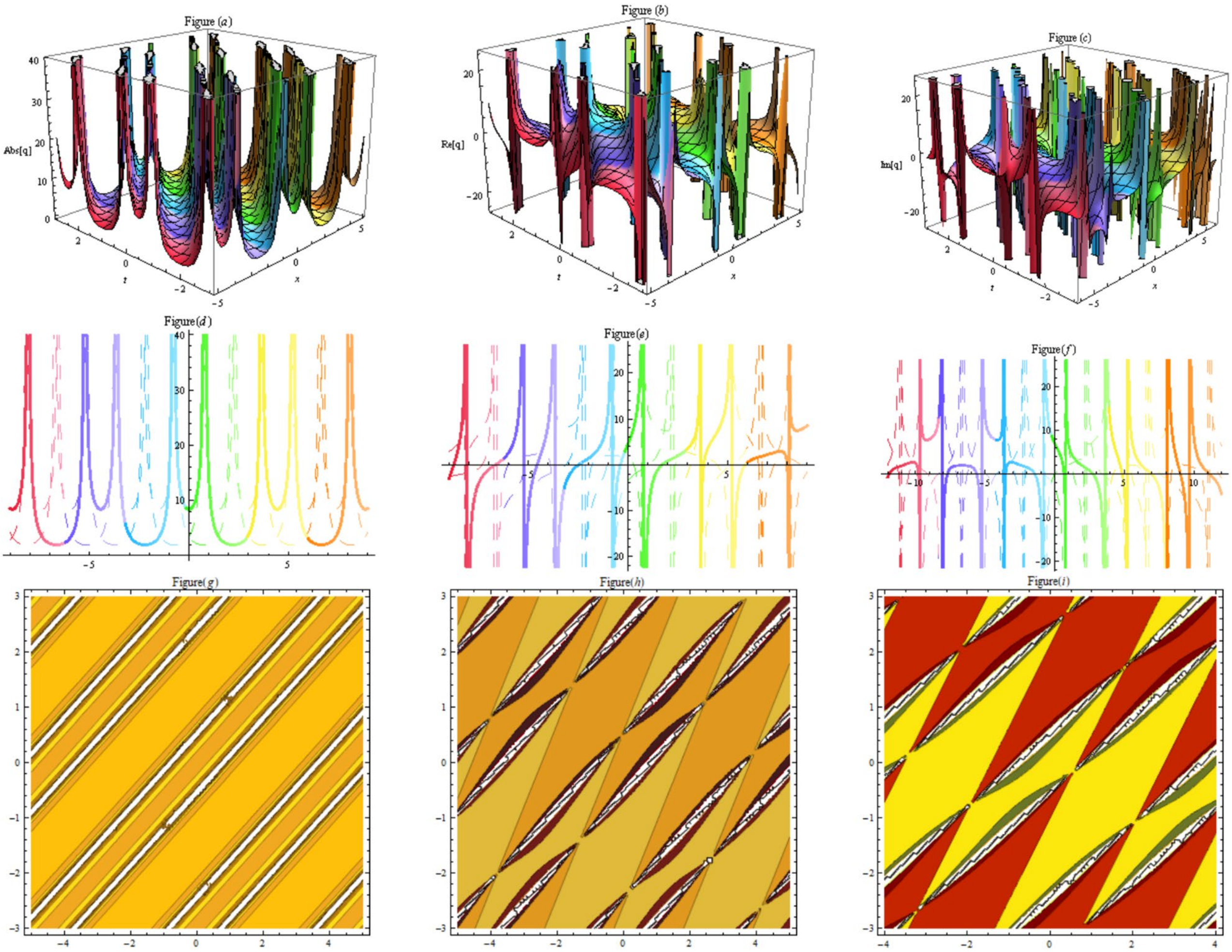

Figure 13. Visualization of periodic wave soliton structure in three different kinds of plotting of $q_8(x,t)$ function by utilizing these values $\nabla_0 = -2, \nabla_1 = 4, \nabla_2 = 3, \kappa = 1, \wp = 1.5, \mu = 1, \Lambda = 1, \epsilon = 1, \Xi = 1, \varpi = 1, b_0 = 1.$

Data Availability

All data that support the findings of this study are included in the article.

Received: 27 August 2025; Accepted: 26 September 2025
Published online: 03 November 2025

References

1. Abdalla, M., Roshid, M. M., Ullah, M. S. & Hossain, I. Dynamical analysis, and the effect of fractional parameters on optical soliton solution for Yajima-Oikawa model in short-wave and long-wave. *Chaos Solitons Fractals* **199**, 116697 (2025).
2. Alam, N. et al. Novel dynamics of the fractional KFG equation through the unified and unified solver schemes with stability and multistability analysis. *Nonlinear Eng.* **13**(1), 20240034 (2024).
3. Han, T., Rezazadeh, H. & Rahman, M. U. High-order solitary waves, fission, hybrid waves and interaction solutions in the nonlinear dissipative (2+ 1)-dimensional Zabolotskaya-Khokhlov model. *Phys. Scr.* **99**(11), 115212 (2024).
4. Iqbal, M. et al. Analysis of periodic wave soliton structure for the wave propagation in nonlinear low-pass electrical transmission lines through analytical technique. *Ain Shams Eng. J.* **16**(9), 103506 (2025).
5. Zhou, G. et al. Off-axis four-reflection optical structure for lightweight single-band bathymetric LiDAR. *IEEE Trans. Geosci. Remote Sens.* **61**, 1–17 (2023).
6. Rahaman, M. S., Islam, M. N. & Ullah, M. S. Bifurcation, chaos, modulation instability, and soliton analysis of the Schrödinger equation with cubic nonlinearity. *Sci. Rep.* **15**(1), 11689 (2025).
7. Wang, Y. et al. Non-interleaved shared-aperture full-stokes metalens via prior-knowledge-driven inverse design. *Adv. Mater.* **37**(8), 2408978 (2025).
8. Luo, K. et al. Study of polarization transmission characteristics in nonspherical media. *Opt. Lasers Eng.* **174**, 107970 (2024).
9. Iqbal, M. et al. Nonlinear behavior of dispersive solitary wave solutions for the propagation of shock waves in the nonlinear coupled system of equations. *Sci. Rep.* **15**(1), 27535 (2025).
10. Hao, Y. et al. Characteristics of carrier localization and their effects on minority carrier lifetime in InAs/In0.5Ga0.5As0.5Sb0.5 type II superlattices. *Appl. Phys. Lett.* **127**(8), 82105 (2025).
11. Zahed, H., Seadawy, A. R. & Iqbal, M. Structure of analytical ion-acoustic solitary wave solutions for the dynamical system of nonlinear wave propagation. *Open Phys.* **20**(1), 313–333 (2022).
12. Iqbal, M. et al. Exploring the peakon solitons molecules and solitary wave structure to the nonlinear damped Kortewege-de Vries equation through efficient technique. *Open Phys.* **23**(1), 20250198 (2025).
13. Wang, Y. et al. High numerical aperture achromatic meta-devices through dispersion compensation. *Adv. Funct. Mater.* **2025**, e15507 (2025).
14. Wang, Z. et al. Wave propagation in finite discrete chains unravelled by virtual measurement of dispersion properties. *IET Sci. Meas. Technol.* **18**(6), 280–288 (2024).
15. Seadawy, A. R., Zahed, H. & Iqbal, M. Solitary wave solutions for the higher dimensional Jimo-Miwa dynamical equation via new mathematical techniques. *Mathematics* **10**(7), 1011 (2022).
16. Hussein, H. H. et al. Multiple soliton solutions and other travelling wave solutions to new structured (2+ 1)-dimensional integro-partial differential equation using efficient technique. *Phys. Scr.* **99**(10), 105270 (2024).
17. Gao, N., Guo, X., Deng, J., Cheng, B. & Hou, H. Elastic wave modulation of double-leaf ABH beam embedded mass oscillator. *Appl. Acoust.* **173**, 107694 (2021).
18. Dong, C. et al. The model and characteristics of polarized light transmission applicable to polydispersity particle underwater environment. *Opt. Lasers Eng.* **182**, 108449 (2024).
19. Samir, I., Ahmed, K. K., Ahmed, H. M., Emadifar, H. & Rabie, W. B. Extraction of newly soliton wave structure of generalized stochastic NLSE with standard Brownian motion, quintuple power law of nonlinearity and nonlinear chromatic dispersion. *Phys. Open* **21**, 100232 (2024).
20. Usman, T., Hossain, I., Ullah, M. S. & Hasan, M. M. Soliton, multistability, and chaotic dynamics of the higher-order nonlinear Schrödinger equation. *Chaos* **35**(4), 6469 (2025).
21. Han, T., Liang, Y. & Fan, W. Dynamics and soliton solutions of the perturbed Schrödinger-Hirota equation with cubic-quintic-septic nonlinearity in dispersive media. *AIMS Math.* **10**(1), 754–776 (2025).
22. Iqbal, M. et al. On the exploration of dynamical optical solitons to the modify unstable nonlinear Schrödinger equation arising in optical fibers. *Opt. Quant. Electron.* **56**(5), 765 (2024).
23. Mostafa, M. & Ullah, M. S. Soliton outcomes and dynamical properties of the fractional Phi-4 equation. *AIP Adv.* **15**(1), 015105 (2025).
24. Alam, N. et al. Bifurcation analysis, chaotic behaviors, and explicit solutions for a fractional two-mode Nizhnik–Novikov–Veselov equation in mathematical physics. *AIMS Math.* **10**(3), 4558–4578 (2025).
25. Iqbal, M., Seadawy, A. R., Khalil, O. H. & Lu, D. Propagation of long internal waves in density stratified ocean for the (2+ 1)-dimensional nonlinear Nizhnik-Novikov-Vesselov dynamical equation. *Results Phys.* **16**, 102838 (2020).
26. Ganie, A. H., Rahaman, M. S., Aladsani, F. A. & Ullah, M. S. Bifurcation, chaos, and soliton analysis of the Manakov equation. *Nonlinear Dyn.* **113**(9), 9807–9821 (2025).
27. Iqbal, M. et al. Exploring the optical soliton and solitary wave solutions for the nonlinear Akbota equation via improved expansion approach. *AIP Adv.* **15**(9), 015101 (2025).
28. Iqbal, M., Seadway, A. R., Lu, D. & Zhang, Z. Computational and analytical techniques for long dispersive wave and construction of solitary wave solutions for nonlinear Whitham-Broer-Kaup equation. *J. Appl. Nonlinear Dyn.* **14**(01), 211–232 (2025).
29. Iqbal, M., Seadawy, A. R., Lu, D. & Zhang, Z. Weakly restoring forces and shallow water waves with dynamical analysis of periodic singular solitons structures to the nonlinear Kadomtsev-Petviashvili-modified equal width equation. *Mod. Phys. Lett. B* **38**(27), 2450265 (2024).
30. Alammari, M. et al. Exploring the nonlinear behavior of solitary wave structure to the integrable Kairat-X equation. *AIP Adv.* **14**(11), 115006 (2024).
31. Murad, M. A. S., Mahmood, S. S., Emadifar, H., Mohammed, W. W. & Ahmed, K. K. Optical soliton solution for dual-mode time-fractional nonlinear Schrödinger equation by generalized exponential rational function method. *Results Eng.* **27**, 105591 (2025).
32. Ahmed, K. K. et al. Characterizing stochastic solitons behavior in (3+ 1)-dimensional Schrödinger equation with Cubic-Quintic nonlinearity using improved modified extended tanh-function scheme. *Phys. Open* **21**, 100233 (2024).
33. Alam, M. N. et al. Bifurcation, phase plane analysis and exact soliton solutions in the nonlinear Schrodinger equation with Atangana's conformable derivative. *Chaos Solitons Fractals* **182**, 114724 (2024).
34. Iqbal, M. et al. A construction of novel soliton solutions to the nonlinear fractional Kairat-II equation through computational simulation. *Opt. Quant. Electron.* **56**(5), 845 (2024).
35. Han, T., Zhang, K., Jiang, Y. & Rezazadeh, H. Chaotic pattern and solitary solutions for the (21)-dimensional beta-fractional double-chain DNA system. *Fractal Fract.* **8**(7), 415 (2024).
36. Han, T., Jiang, Y. & Lyu, J. Chaotic behavior and optical soliton for the concatenated model arising in optical communication. *Results Phys.* **58**, 107467 (2024).

37. Iqbal, M., Lu, D., Seadawy, A. R. & Zhang, Z. Nonlinear behavior of dust acoustic periodic soliton structures of nonlinear damped modified Korteweg-de Vries equation in dusty plasma. *Results Phys.* **59**, 107533 (2024).
38. Iqbal, M. et al. Exploration of unexpected optical mixed, singular, periodic and other soliton structure to the complex nonlinear Kuralay-IIA equation. *Optik* **301**, 171694 (2024).
39. Alshehery, S. et al. Construction of periodic wave soliton solutions for the nonlinear Zakharov-Kuznetsov modified equal width dynamical equation. *Opt. Quant. Electron.* **56**(8), 1381 (2024).
40. Murad, M. A. S. et al. Analysis of Kudryashov's equation with conformable derivative via the modified Sardar sub-equation algorithm. *Results Phys.* **60**, 107678 (2024).
41. Az-Zo'bi, E. A. et al. Chaotic, bifurcation, sensitivity, modulation stability analysis, and optical soliton structure to the nonlinear coupled Konno-Oono system in magnetic field. *AIP Adv.* **15**(9), 0915103 (2025).
42. Jazaa, Y. et al. On the exploration of solitary wave structures to the nonlinear Landau-Ginsberg-Higgs equation under improved F-expansion method. *Opt. Quant. Electron.* **56**(7), 1181 (2024).
43. Alqahtani, S. et al. Analysis of mixed soliton solutions for the nonlinear Fisher and diffusion dynamical equations under explicit approach. *Opt. Quant. Electron.* **56**(4), 647 (2024).
44. Murad, M. A. S., Faridi, W. A., Jhangeer, A., Iqbal, M. & Garayev, M. Optical Solutions With Kudryashov â€$^{\text{TM}}$ S Arbitrary Type Of Generalized Non-Local Nonlinearity And Refractive Index Via Kudryashov Auxiliary Equation Method. *FRACTALS (fractals)***33**(04), 1–15 (2025).
45. Seadawy, A. R. & Iqbal, M. Optical soliton solutions for nonlinear complex Ginzburg-Landau dynamical equation with laws of nonlinearity Kerr law media. *Int. J. Mod. Phys. B* **34**(19), 2050179 (2020).
46. Iqbal, M. & Seadawy, A. R. Instability of modulation wave train and disturbance of time period in slightly stable media for unstable nonlinear Schrödinger dynamical equation. *Mod. Phys. Lett. B* **34**(supp01), 2150010 (2020).
47. Faridi, W. A., Iqbal, M. & Mahmoud, H. A. An invariant optical soliton wave study on integrable model: A Riccati-Bernoulli sub-optimal differential equation approach. *Int. J. Theor. Phys.* **64**(3), 1–23 (2025).
48. Alruwaili, A. D., Seadawy, A. R., Iqbal, M. & Beinane, S. A. O. Dust-acoustic solitary wave solutions for mixed nonlinearity modified Korteweg-de Vries dynamical equation via analytical mathematical methods. *J. Geom. Phys.* **176**, 104504 (2022).
49. Seadawy, A. R. & Iqbal, M. Dispersive propagation of optical solitions and solitary wave solutions of Kundu-Eckhaus dynamical equation via modified mathematical method. *Appl. Math.* **38**(1), 16–26 (2023).
50. Shakeel, M. & Liu, X. Solitary wave solutions, bifurcation theory, and sensitivity analysis in the modified unstable nonlinear Schrödinger equation. *Mod. Phys. Lett. B* **39**(22), 2550082 (2025).
51. Huang, Z. et al. Piecewise calculation scheme for the unconditionally stable Chebyshev finite-difference time-domain method. *IEEE Trans. Microw. Theory Tech.* **73**(8), 4588–4596 (2025).
52. Faridi, W. A., Tipu, G. H., Myrzakulova, Z., Myrzakulov, R. & Akinyemi, L. Formation of optical soliton wave profiles of Shynaray-IIA equation via two improved techniques: A comparative study. *Opt. Quant. Electron.* **56**(1), 132 (2024).
53. Tipu, G. H. et al. The optical exact soliton solutions of Shynaray-IIA equation with Φ^6-model expansion approach. *Opt. Quant. Electron.* **56**(2), 226 (2024).
54. Altalbe, A., Taishiyeva, A., Myrzakulov, R., Bekir, A. & Zaagan, A. A. Effect of truncated M-fractional derivative on the new exact solitons to the Shynaray-IIA equation and stability analysis. *Results Phys.* **57**, 107422 (2024).
55. Khan, M. I., Marwat, D. N. K., Sabi'u, J. & Inc, M. Exact solutions of Shynaray-IIA equation (S-IIAE) using the improved modified Sardar sub-equation method. *Opt. Quant. Electron.* **56**(3), 459 (2024).
56. Amer, A., Boulaaras, S. M., Althobaiti, A., Althobaiti, S. & Ur Rehman, H. Unveiling soliton dynamics: Innovative methods and numerical insights for the Shynaray-IIA equation. *Int. J. Mod. Phys. B* **39**, 2550203 (2025).
57. Iqbal, M. et al. Dynamical analysis of soliton structures for the nonlinear third-order Klein-Fock-Gordon equation under explicit approach. *Opt. Quant. Electron.* **56**(4), 651 (2024).
58. Iqbal, M., Lu, D., Faridi, W. A., Murad, M. A. S. & Seadawy, A. R. A novel investigation on propagation of envelop optical soliton structure through a dispersive medium in the nonlinear Whitham-Broer-Kaup dynamical equation. *Int. J. Theor. Phys.* **63**(5), 131 (2024).

Acknowledgements
The authors extend their appreciation to King Saud University, Saudi Arabia for funding this work through On-going Research Funding Program, (ORF-2025-305), King Saud University, Riyadh, Saudi Arabia.

Author contributions
MI: Writing-Original draft preparation, Methodology, Conceptualization. WAF: Visualization, Writing-Reviewing & Editing. RA: Visualization, Validation. AA: Software, Data curation. MEM: Formal analysis, Writing-Reviewing & Editing. NM: Visualization, Supervision. KAA: Resources, Investigation.

Funding
Not applicable.

Declarations

Conflicts of Interest
The authors declare no conflict of interest.

Competing interests
The authors declare no competing interests.

Additional information
Correspondence and requests for materials should be addressed to N.M.