On the Two-Fold Maximal Units in Some Two-Fold Finite Neutrosophic Rings Modulo Integers for ≤ ≤

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Abstract

In this paper, we have defined the concept of twofold maximal units in finite twofold neutrosophic rings modulo integers, where a sufficient and necessary condition for such class of generalized units will be provided. We characterize all maximal units in the following twofold neutrosophic rings (()) for ∊ {2,3,4,5}.

On the Two-Fold Maximal Units in Some Two-Fold Finite Neutrosophic Rings Modulo Integers for $2 \le n \le 5$

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Abstract

In this paper, we have defined the concept of two-fold maximal units in finite two-fold neutrosophic rings modulo integers, where a sufficient and necessary condition for such class of generalized units will be provided. We characterize all maximal units in the following two-fold neutrosophic rings $(Z_n(I))_{f_I}$ for $n \in \{2,3,4,5\}$.

Keywords: Two-fold algebra; Finite neutrosophic ring; Modulo integer ring; minimal unit

1. Introduction

The question of determining the units in an algebraic ring has attracted the attention of many researchers, specifically in some modern rings such as neutrosophic rings, and n-cyclic refined neutrosophic rings [6-8], where we find a classification of the group of units in several special solutions, and also tables that calculate the value of these units and determine their exact number [9-20].

A unit in a ring R is the concept of being an invertible element concerning multiplication operation, which make a group together. Two-fold neutrosophic algebras are new algebraic structures presented by Smarandache [1] by combining neutrosophic values of truth, falsity, and indeterminacy with classical algebraic sets. Many authors to generalize other famous algebraic structures such as two-fold fuzzy number theoretical systems [2-3], two-fold modules and spaces [4], and two-fold fuzzy rings [5] used these ideas.

In this work, we define the maximal two-fold neutrosophic finite ring modulo integers, and we determine all maximal units in these rings for the special values of n between 2 and 5. We have also classified all the units that have been calculated in tables showing their values as well as their number.

2. Main Discussion

Definition 2.1:

Let $Z_n = \{0,1,\dots,n-1\}$ be the ring of integers modulo n, and $Z_n(I) = \{a + bI \,; I^2 = I \,, a, b \in Z_n\}$ be the corresponding neutrosophic ring. Assume that $f: Z_n \to [0,1]$ be a fuzzy mapping with

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$\begin{cases} f(0) = 0 \\ f(1) = 1 \end{cases}$; $f_I: Z_n(I) \to [0,1]$ be the mapping defined as follows:

$f_I(a+bI) = \max(f(a), f(b))$.

The maximal two-fold neutrosophic ring$(Z_n(I))_{f_I}$ as follows:

$(Z_n(I))_{f_I} = \{(a+bI)_{f_I(c+dI)}\ ;\ a+bI, c+dI \in Z_n(I)\}$.

Example 2.1:

Consider $(Z_5 = \{0,1,2,3,4\}, +, \cdot)$ the ring of integers modulo 5, take $f : Z_5 \to [0,1]$;

$$f(x) = \begin{cases} 1\ ; x = 1 \\ \frac{1}{2}\ ; x \in \{2,3\} \\ \frac{1}{4}\ ; x = 4 \\ 0; x = 0 \end{cases},$$

$Z_5(I) = \{0,1,2,3,4, I, 2I, 3I, 4I, 1+I, 1+2I, 1+3I, 1+4I, 2+I, 2+2I, 2+3I, 2+4I, 3+I, 3+2I, 3+3I, 3+4I, 4+I, 4+2I, 4+3I, 4+4I\}$.

$f_I(3+4I) = \max(f(3), f(4)) = \frac{1}{2}, f_I(1+2I) = max(f(1), f(2)) = 1$. By a similar approach, we can see:

$f_I(1+4I) = 1, f_I(2+4I) = \frac{1}{2}, f_I(0) = 0, f_I(I) = 1, f_I(2I) = \frac{1}{2}, f_I(3I) = \frac{1}{2}, f_I(4I) = \frac{1}{4}, f_I(1+3I) = 1,$

$f_I(2+I) = 1, f_I(3+I) = 1, f_I(2) = f_I(3) = 1\backslash 2$, and so on.

Therefore, $(Z_5(I))_{f_I} = \{(a+bI)_0, (a+bI)_1, (a+bI)_{\frac{1}{2}}, (a+bI)_{\frac{1}{4}}; for\ all\ a, b \in Z_5\}$

Definition 2 .2

Let $(Z_n(I))_{f_I}$ be the maximal two-fold neutrosophic ring, then$(a+bI)_{f_I(m+nI)}$ is called a maximal unit if and only if there exists:

$(c+dI)_{f_I(t+kI)} \in (Z_n(I))_{f_I}$ such that:

$(a+bI)_{f_I(m+nI)} \circ (c+dI)_{f_I(t+kI)} = 1_1$.

Remark 2.1

The operation $(\circ)$: $Z_n(I) \times Z_n(I) \to Z_n(I)$ is defined as:

$(a+bI)_{f_I(m+nI)} \circ (c+dI)_{f_I(t+kI)} = (ac + (ad+bc+bd)I)_{f_I(mt+(mk+nt+nk)I)}$

Example 2.2

Consider $Z_3 = \{0,1,2\}, Z_3(I) = \{0,1,2, I, 2I, 1+I, 1+2I, 2+I, 2+2I\}$,

Consider $f: Z_3 \to [0,1]$; $\begin{cases} f(0) = 0 \\ f(1) = 1 \\ f(2) = \frac{1}{2} \end{cases}$

Then: $f_I: Z_3(I) \to [0,1]$ such that:

$f_I(0) = 0, f_I(1) = 1, f_I(2) = \frac{1}{2}, f_I(I) = 1, f_I(2I) = \frac{1}{2}, f_I(1+I) = 1, f_I(1+2I) = 1, f_I(2+I) = 1, f_I(2+2I) = \frac{1}{2}$.

So that $(Z_3(I))_{f_I} = \{0_0, 0_{\frac{1}{2}}, 0_1, 1_0, 1_1, 1_{\frac{1}{2}}, I_0, I_1, I_{\frac{1}{2}}, 2_0, 2_1, 2_{\frac{1}{2}}, (2I)_0, (2I)_1, (2I)_{\frac{1}{2}}, (1+I)_0, (1+I)_1, (1+I)_{\frac{1}{2}}, (1+2I)_1, (2+I)_1, (2+2I)_{\frac{1}{2}}, (2+I)_0, (2+I)_{\frac{1}{2}}, (1+2I)_0, (1+2I)_{\frac{1}{2}}, (2+2I)_0, (2+2I)_1\}$.

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