Ionics
https://doi.org/10.1007/s11581-024-05999-z

RESEARCH

Accurate optimizing proton exchange membrane fuel cell parameters using fitness deviation-based adaptive differential evolution

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Received: 28 October 2024 / Revised: 24 November 2024 / Accepted: 3 December 2024

Abstract

Proton exchange membrane fuel cells (PEMFCs) are complex, nonlinear systems whose performance depends on several interrelated parameters. Accurate estimation of these parameters is crucial for enhancing the efficiency and reliability of PEMFCs. In this paper, we used Fitness Deviation-based Differential Evolution (FD-DE) algorithm to optimally identify the unknown parameters of PEMFC models. An adaptive parameter control with wavelet basis function and Gaussian distribution, a hybrid trial vector generation strategy using t-distribution based perturbation, and a dimensional replacement mechanism for maintaining population diversity are introduced in the FD-DE algorithm. The innovations in these algorithms tackle the common problems in differential evolution algorithms, including premature convergence and loss of diversity. The proposed FD-DE algorithm is validated on twelve different PEMFC case studies under different operating conditions and compared with several state-of-the-art algorithms, including other DE variants and non-DE algorithms. For that purpose, the optimization targets seven parameters $\xi_1, \xi_2, \xi_3, \xi_4, \lambda, R_c$ and B to closely match the polarization curves to those specified in the manufacturer datasheet, focusing initially on six primary stacks: BCS 500-W PEM, STD 250-W PEM, Nedstack PS6 PEM, 500W SR-12PEM, H-12 PEM, and HORIZON 500W PWM. The optimization approach is to minimize the sum of squared errors (SSE) between the predicted stack voltages of the model and the experimentally measured results. Absolute error (AE), relative error (RE), and mean bias error (MBE) are assessed across different datasheets. A comparative statistical analysis is made among several DE variants FD-DE, TDE, MadDE, LSHADE, LSHADE-cnEpSin, jSO, PaDE, and non-DE algorithms ACS, SSA, and TEO. The results show that the FD-DE algorithm performs consistently better than existing strategies in terms of accuracy, convergence speed and stability, and provides better parameter estimation with lower errors. The effectiveness of the FD-DE algorithm in solving complex optimization problems and its potential to enhance PEMFC modeling and analysis is shown in this work.

Keywords Parameter estimation · Proton exchange membrane fuel cell · PEMFC · Differential evolution · Optimization

Abbreviations

AE	Absolute Error
ACS	Ant Colony System
CR	Crossover Rate
DE	Differential Evolution
FD-DE	Fitness Deviation-based Differential Evolution
GA	Genetic Algorithm
SHADE	Success-history based parameter adaptation for differential evolution
LPalmDE	Laplace Palm Differential Evolution
LSHADE	SHADE with Linear Population Size Reduction
MBE	Mean Bias Error
MadDE	Modified Adaptive Differential Evolution
nfe	Number of Function Evaluations
nfe_max	Maximum Number of Function Evaluations
PaDE	Parameter Adaptive Differential Evolution
PEMFC	Proton Exchange Membrane Fuel Cell
PS	Population Size
PS_max	Maximum Population Size
PS_min	Minimum Population Size
PSO	Particle Swarm Optimization
RE	Relative Error
RT	Runtime
SSE	Sum of Squared Errors
SSA	Salp Swarm Algorithm
TEO	Thermal Exchange Optimization

Extended author information available on the last page of the article

Published online: 23 December 2024

tpdf	T-distribution Probability Density Function
F	Scale Factor
CR	Crossover Rate

Introduction

Recently, renewable energy sources (RESs) have been the most attractive alternatives to fossil fuels, which are the major source of clean energy because of its non-polluting nature. Fuel cells (FCs) are one of the most important technologies in the renewable energy spectrum and are being used more and more as backup power sources to meet the growing load demands. A fuel cell is a device that converts chemical reactions into electrical energy by means of a negative anode, positive cathode and electrolyte. Fuel cells are differentiated by the type of electrolyte used and start up time, with proton exchange membrane (PEM) fuel cells in particular being preferred for their efficiency.

Modeling PEMFCs is difficult due to nonlinear behavior and sensitivity to operating conditions (such as temperature and pressure). Some critical parameters are not specified in the manufacturing datasheets, and accurate identification is required to build a realistic PEMFC model in many cases. Running a fuel cell under suboptimal conditions can decrease performance and shorten its lifetime, reducing operational life by orders of magnitude. Hence, it is critical to understand how each one of the various operating parameters affect PEMFC performance. Past studies that have attempted to identify optimal parameters for fuel cells will also be reviewed in this paper, and the ongoing need for accurate parameter estimation will be discussed.

Reliable PEMFC modelling relies on experimental data, which is typically segmented into three categories: There are three types of models: data-driven models [1, 2], empirical formula-based models [3–5], and hybrid models [6]. The most popular data-driven models for capturing the relationship between input and output variables are neural network (NN) algorithms. For example, an artificial neural network (ANN) was used to develop a data driven model to predict the output performance of high temperature PEMFC systems [7]. This model was trained using numerical simulation data.

In the past few years, a great deal of research has been focusing on identifying, improving, and fine tuning the unknown parameters of semi empirical PEMFC system models. In this context, intelligent optimization techniques, in particular, metaheuristic methods, have been successfully used. This problem has been tackled using traditional optimization algorithms, standard DE [8, 9] such as Genetic Algorithms (GA) [10, 11] and Particle Swarm Optimization (PSO) [12–14]. Nevertheless, these algorithms can be plagued by complex, high dimensional search spaces and can prematurely converge to local optima. Additionally, they can be sensitive to initial parameter settings and may not effectively maintain diversity within the population, leading to stagnation. Despite its benefits, the traditional GA often struggles with complex structures, slow convergence, and imprecision in high-dimensional challenges. Consequently, a PSO-based technique was developed for the Nexa 1.2 kW PEMFC model to overcome these limitations [12]. Additionally, a momentum-enhanced PSO demonstrated rapid convergence in accurate PEMFC modeling [13], and a chaos-embedded PSO algorithm was formulated with a novel objective function for realistic PEMFC parameter identification [14]. This approach effectiveness was confirmed through polarization data from three distinct commercial fuel cells, including the 250 W Stack, BCS-500 W stack, and Nedstack PS6.

The performance of PSO algorithms is unfortunately very sensitive to the values of learning and inertia factors. However, fine tuning these parameters is computationally expensive, and incorrect settings can lead to premature convergence problems. As a result, besides the widely used GA and PSO, several novel metaheuristic algorithms have been proposed for parameter identification of semiempirical PEMFC models. Included in this category are: the Shark Smell Optimization (SSO) method [15], Golden Jackal Optimization (GJO) algorithm [16], and Grey Wolf Optimization (GWO) algorithm [17]. The SSO method was used to identify the unknown parameters of the semi empirical model in [15] and then validated against polarization test data from five commercial fuel cell stacks. In [16], a PSO based GJO method was developed to minimize the sum of squared errors (SSE) between the measured output voltage and the output voltage of the PEMFC stack. It was shown that it optimally estimates the PEMFC model parameters better than other methods.

An optimal method to estimate PEMFC parameters is proposed by Yin and Razmjooy [18] based on the Deer Hunting Optimization (DHO) algorithm. The DHO algorithm has been used to improve the PEMFC parameters identification. An Adaptive Neuro Fuzzy Inference System (ANFIS) was used by Yang et al. [19] to model a 250 W PEMFC mounted on an electric bicycle. For determining the system configuration, inputs included humidity, temperature, hydrogen, oxygen flow rates, and current, while efficiency and voltage were considered outputs. Simulation results demonstrated that using ANFIS for PEMFC modeling yields reliable and accurate predictions of PEMFC performance. Although classical methods and ANN provided logical results, the introduction of metaheuristics attracted researchers due to their simplicity and time-saving procedures in resolving modeling problems.

The Simulink library in MATLAB was used by Abdin et al. [20] to develop an advanced model for PEMFC. An algorithm named AC-POA [21] was used by Yang et al.

Table 1 Comparison of literature on PEMFC parameter estimation

Algorithm Used	Key Contribution	Strengths	Limitations
Particle Swarm Optimization (PSO) [12]	Parameter identification for N-1.2 kW PEMFC	Rapid convergence	Sensitive to learning and inertia factors
Momentum-Enhanced PSO [13]	Rapid convergence in PEMFC modeling	Improved convergence speed	High computational cost
Chaos-Embedded PSO [14]	Accurate PEMFC parameter identification	Overcame local optima issues	Fine-tuning required
Shark Smell Optimization (SSO) [15]	Identified unknown parameters of PEMFC models	Effective in avoiding local optima	High computational complexity
Golden Jackal Optimization (GJO) [16]	Minimized SSE for PEMFC model parameters	Outperformed other methods in parameter estimation	Sensitive to initial settings
Grey Wolf Optimization (GWO) [17]	Effective parameters identification	High convergence precision	Computationally expensive
Deer Hunting Optimization (DHO) [18]	Enhanced identification of PEMFC parameters	Improved exploration–exploitation balance	Limited scalability for large problems
Adaptive Neuro-Fuzzy Inference System (ANFIS) [19]	Accurate predictions for PEMFC performance	Highly reliable for small datasets	Requires extensive training data
Nature-Inspired Algorithms [24]	Optimized PEMFC design and parameters	Overcame limitations of GA	Computationally expensive
Honey Badger Optimizer (HBO) [25]	Improved parameter extraction accuracy	Balanced exploration and exploitation	Sensitive to hyperparameters
Enhanced Bald Eagle Algorithm (EBEA) [27]	Optimized PEMFC models	Improved convergence speed	High computational cost
Autonomous Groups Particle Swarm Optimization (AGPSO) [28]	Balanced accuracy and efficiency in PEMFC modeling	Computationally efficient	Limited scalability
Chaotically-Based Bonobo Optimizer (CBO) [29]	Enhanced exploration and exploitation	Improved accuracy in PEMFC parameter estimation	Sensitive to chaotic variables
Whale Optimization Algorithm (WOA) [30]	Efficient and accurate PEMFC modeling	Adaptive learning scheme for parameter control	Struggles with large-scale problems
Social Learning-Based Optimization (SLO) [31]	Reduced function evaluations while maintaining accuracy	Efficient for parameter estimation	Scalability issues in complex problems
Dandelion Optimization Algorithm (DOA) [33]	Robust performance in high-dimensional search spaces	Effective in PEMFC parameter estimation	High computational complexity
Standard and Quasi-Oppositional Bonobo Optimizers (BO) [35]	Improved convergence speed and accuracy	Effective for PEMFC parameter extraction	Sensitive to problem-specific tuning
FD-DE [37]	Enhanced PEMFC parameter estimation	Solved premature convergence and diversity issues	Computationally efficient and scalable

to estimate the PEMFC terminal voltage. However, some parameters are not given in the PEMFC datasheet provided by the manufacturer, so they must be estimated to complete and accurately model the PEMFC. The main goal of this work is to find the best values of these PEMFC parameters to achieve polarization curves as close as possible to experimental data. These parameters are acquired, enabling the creation of a complete mathematical model for PEMFCs. The optimization process is to minimize a proposed fitness function, which is the sum of squared differences between the experimental stack voltage and calculated stack voltage. An optimization model for high power PEMFC was proposed by Zhou et al. [22] and solved by improved Grey Wolf Optimizer. Metaheuristic algorithms are increasingly being used to optimize the parameter of PEMFCs. There have been numerous recent studies that investigate various optimization techniques to tackle the problem of identifying unknown parameters in Proton Exchange Membrane Fuel Cells (PEMFCs). A gradient based optimizer for PEMFC parameter estimation was proposed by Rezk et al. [23], which has the advantage of providing accurate results at relatively low computational cost. Nevertheless, this method is sensitive to initial conditions and is not suitable for complex scenarios. Nature inspired algorithms were used by Blanco-Cocom et al. [24] to optimize the design and parameter estimation of PEMFCs. Evolutionary strategy-based algorithms are shown to overcome the limitations of traditional methods such as genetic algorithms which are prone to local optima and slow convergence. Likewise, Ashraf et al. [25] presented the Honey Badger Optimizer (HBO) to extract unknown parameters of PEMFC models, which improved balancing of exploration and exploitation and avoided local optima traps and more accurate parameter estimation.

Similar to Rezk et al. [26], fuzzy modelling, coupled with optimization techniques, was also used to determine the optimal parameters for PEMFCs. The parameter identification process was improved by this approach, and it showed the flexibility of fuzzy systems in dealing with uncertainties. Enhanced Bald Eagle Algorithm (EBEA) was applied by Alsaidan et al. [27] to optimize PEMFC models, in order to improve convergence speed and reduce computational complexity to achieve better performance in identifying key PEMFC parameters. The Gazelle Optimization Algorithm (GOA) proposed by Haddad et al. [28] for PEMFC parameter optimization, converges faster and with higher precision, and is applicable to complex multi-dimensional problems. Far et al. [29] used the Autonomous Groups Particle Swarm Optimization (AGPSO) algorithm to determine optimal PEMFC parameters, with a balanced trade-off between accuracy and computational efficiency.

An event triggered and dimension learning scheme using the Whale Optimization Algorithm (WOA) for PEMFC modelling was introduced by Sun et al. [30] where the optimization process was dynamically adjusted to produce more efficient and accurate results. Social Learning Based Optimization (SLO) was applied to estimate PEMFC design parameters by Celtek [31], which reduced the number of function evaluations without sacrificing accuracy. An improved Grey Wolf Optimizer (GWO) based on a neighbourhood trust model for parameter identification is proposed by Zhu et al. [32], which improves exploration and deals with the complexities of PEMFC modelling.

The Dandelion Optimization Algorithm (DOA) introduced by Abbassi et al. [33] for estimating key PEMFC parameters showed robust performance in high dimensional search space and in parameter estimation tasks. Chaotically based Bonobo Optimizer (CBO) [34] used chaos theory to improve the exploration capabilities of the algorithm and achieved better identification accuracy. Standard and Quasi-Oppositional Bonobo Optimizers (BO) were applied by Sultan et al. [35] for parameter extraction in PEMFC stacks, and they demonstrated improvements in convergence speed and accuracy. Lastly, Han et al. [36] proposed Chaotic Binary Shark Smell Optimizer (CBSSO) to optimize PEMFC parameters by using chaotic mechanism to escape local optima and obtain more accurate results.

Although these optimization techniques have brought much advancement, they have a few limitations. However, many algorithms, including gradient based methods, are still sensitive to initial conditions leading to suboptimal solutions. However, nature inspired algorithms are often limited by high computational complexity resulting from population-based searches, which make them unsuitable

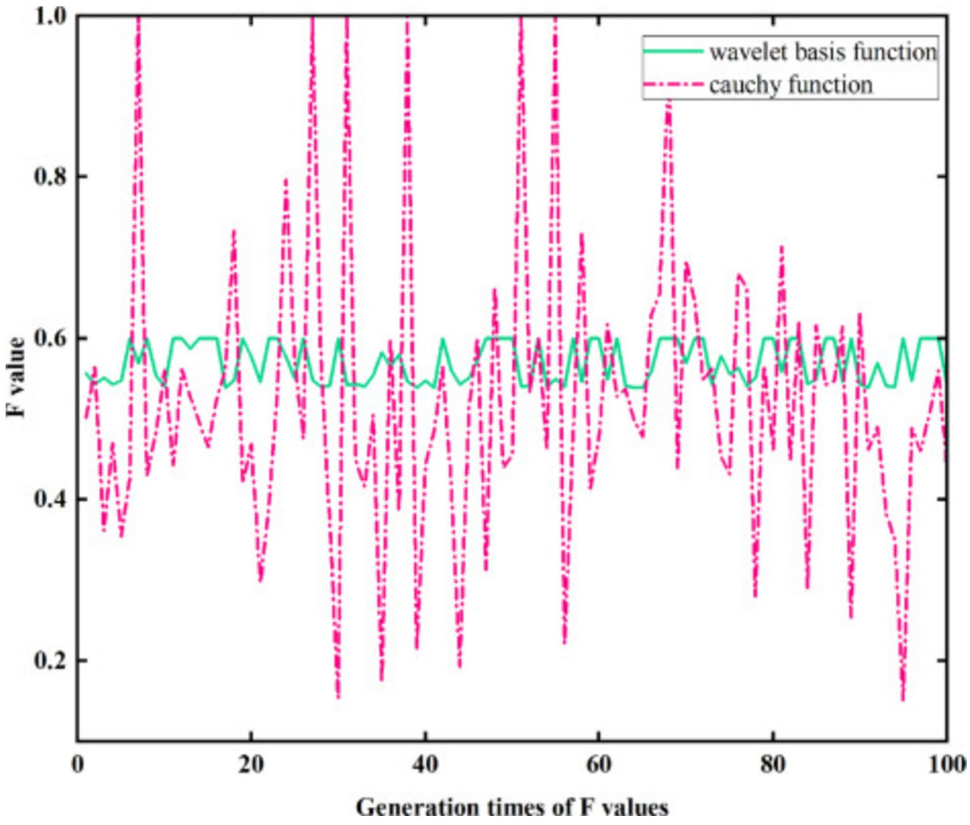


Fig. 1 The generation comparison of *F* by employing Cauchy function and Wavelet basis function [37]

Table 2 Characteristics of twelves PEMFCs used in this work

PEMFC Type	Power(W)	Ncells (no)	A(cm^2)	l(um)	T(K)	Jmax(mA/cm^2)	PH_2(bar)	PO_2(bar)
BCS 500 W	500	32	64	178	333	469	1.0	0.2095
NetStack PS6	6000	65	240	178	343	1125	1.0	1.0
SR-12	500	48	62.5	25	323	672	1.47628	0.2095
H-12–1	12	13	8.1	25	323	246.9	0.4935	1.0
Ballard Mark V	5000	35	232	178	343	1500	1.0	1.0
STD –1	250	24	27	127	343	860	1.0	1.0
Horizon	500	36	52	25	338	446	0.55	1.0

Table 3 Default parameter settings of the DE and non-DE algorithms in the comparison

Algorithms	Default settings
TDE [49]	$F=0.5$, CR = 0.8, NP = $2D$ and $M_t=0.42$
LSHADE [5]	$\mu_F = 0.5, \mu_{CR} = 0.5, F\&CR$ same as JADE, NP $= 18 \cdot D \sim 4, r^{\mathrm{rac}} = 2.6, p = 0.11, H = 6$
LSHADE-cnEpSin [51]	$\mu_F = 0.5, \mu_{CR} = 0.5$ NP $= 18 \cdot D \sim 4, p2 = 0.4, ps = 0.5, H = 5$
jSO [52]	F, CR, r^{rac} same as iLSHADE,$\mu_F = 0.3, \mu_{CR} = 0.8, NP = 25 \cdot \ln D \cdot \sqrt{D} \sim 4, p = 0.25 \sim 0.125, H = 5$
PaDE [53]	$\mu_F = 0.8, \mu_{Cr} = 0.6, F\&CR$ same as LSHADE,$k = 4, p = 0.11, PS = 25\log(D)\sqrt{D} \sim 4, r^{arc} = 1.6, T_0 = 70, r^d = 0.04$
ACS [54]	m $\in$ [4,50], $K_{pher} \in [10,100], q_0 \in [0.1,10], \alpha\epsilon[0.2,1], \beta\epsilon[1,4]$,
SSA [55]	c_2 and c_3 = rand [0,1], NP = 50 to 200
TEO [56]	*c1 ∈{0 or 1}, c2 ∈{0 or 1},pro = Factors of 0.05 from [0,0.5], TM = Integers from [0,10]*
FD-DE [37]	$F = 0.5, CR = 0.8, p = 0.11, k = 0.6, r^{rac} = 1.4, PS = 25 \cdot \ln D\sqrt{D}, H = 4, nfes = D \cdot 10000, n = 0.6 \cdot PS \cdot D, C_n = 0, G = 1, G_{\max} = 3000$

for real-time or large-scale applications. Another challenge involves the trade-off between exploration and exploitation, where over exploration slows down the convergence, and over exploitation can cause premature convergence. In addition, many algorithms demand significant tuning of algorithm specific parameters, which restricts the portability of the algorithms to different PEMFC models or operating conditions. Also, scalability is an issue since algorithms such as WOA [30] and SLO [31] may not work well with larger problem size. Finally, although many algorithms work well in particular conditions, their ability to generalize across different PEMFC models and operating environments is limited, and further research and refinement is needed as shown in Table 1.

Existing methods for parameter estimation, such as classical optimization algorithms and conventional Differential Evolution (DE) variants, often face challenges like premature convergence, sensitivity to control parameter settings, and difficulty in balancing exploration and exploitation. These limitations can lead to suboptimal parameter estimation, affecting the accuracy and reliability of PEMFC models. The following limitations are found in Existing Methods.

- Premature Convergence: Many existing algorithms lack mechanisms to prevent premature convergence, causing them to get trapped in local optima.
- Parameter Sensitivity: The performance of algorithms like GA and PSO is highly dependent on control parameters, which require careful tuning and can be problem-specific.
- Exploration–Exploitation Trade-off: Balancing global exploration and local exploitation remains a challenge, with many algorithms favoring one over the other.
- Diversity Maintenance: Conventional algorithms may not adequately maintain population diversity, reducing their ability to explore the search space effectively.

Differential Evolution (DE) is widely recognized as one of the most powerful stochastic optimization algorithms for various applications; however, even the most advanced DE variants have notable weaknesses. A recent study [37] introduced a robust new DE variant, Differential Evolution with fitness deviation-based adaptation in parameter control (FD-DE), specifically for single-objective numerical optimization. This study presented several key methodologies and outcomes: an enhanced wavelet basis function was proposed to generate the scale factor for each individual during the initial stage of evolution; a hybrid trial vector generation strategy incorporating perturbation and t-distribution was developed to produce different trial vectors at various stages of evolution; a fitness deviation-based parameter control method was proposed for adapting control parameters;

Table 4 Comparative performance metrics of optimization algorithms for FC1

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−0.98401	−0.85320	−1.01047	−0.92676	−0.88555	−1.08573	−1.11278	−0.90090	−1.08178	−1.19332
ξ_2	0.00301	0.00218	0.00276	0.00291	0.00257	0.00365	0.00332	0.00294	0.00355	0.00329
ξ_3	6.454E-05	3.603E-05	4.26E-05	6.856E-05	5.494E-05	8.557E-05	5.946E-05	7.598E-05	8.009E-05	4.184E-05
ξ_4	−0.00018	−0.00019	−0.00019	−0.00019	−0.00019	−0.00019	−0.00019	−0.00019	−0.00018	−0.00019
λ	20.68135	22.29229	23.00000	21.53507	21.22681	20.99853	21.25844	20.88662	16.42376	20.87724
R_c	0.00075	0.00022	0.00028	0.00019	0.00012	0.00012	0.00012	0.00010	0.00038	0.00010
B	0.01360	0.01625	0.01626	0.01621	0.01622	0.01587	0.01626	0.01613	0.01380	0.01613
Min.	0.05501	0.02560	0.02566	0.02595	0.02554	0.02656	0.02554	0.02549	0.08410	0.02549
Max.	0.19249	0.03810	0.04102	0.03361	0.02659	0.03774	0.02614	0.02565	0.32252	0.02550
Mean	0.11849	0.03178	0.03497	0.02858	0.02585	0.03221	0.02584	0.02553	0.16663	0.02549
Std.	0.04580968	0.00423249	0.00584142	0.00244622	0.00034416	0.00424414	0.00018744	5.91796E-05	0.08436626	2.44857E-06
RT	7.73252	7.42391	5.01192	6.21596	11.73585	7.56563	6.18588	6.97512	0.36707	11.74502
FR	9.3	6.2	7.1	5.4	3.6	6.6	4	2.1	9.7	1

and a novel diversity indicator was introduced, along with a restart scheme that can be triggered if the quality of individuals is detected to be poor. The FD-DE algorithm was validated using a comprehensive test suite containing 130 benchmarks from universal test suites for single-objective numerical optimization, demonstrating significant improvements compared to several well-known state-of-the-art DE variants. Additionally, the FD-DE algorithm was tested in real-world optimization applications, with results further confirming its superiority.

Due to their non-polluting nature and sustainability, renewable energy sources (RESs) have become the most attractive alternatives to fossil fuels. Of these technologies, fuel cells (FCs) are a key innovation that are becoming increasingly used as backup power sources to meet growing energy demand. Fuel cell is an electrochemical device that converts chemical energy into electrical energy by means of reactions between a negative anode, a positive cathode and an electrolyte. Among all, Proton Exchange Membrane Fuel Cells (PEMFCs) are highly preferred because of their high efficiency and low operating temperatures. Although PEMFCs have many advantages, they also have many modeling and performance optimization challenges. Their nonlinear behavior and sensitivity to operating conditions (e.g., temperature, pressure, and humidity) is one major issue that complicates the development of accurate models to predict their performance under changing conditions. Such parameters as kinetic, thermodynamic, and electrochemical coefficients are often not available in manufacturer datasheets. Moreover, PEMFC performance is influenced by a number of interdependent parameters such that the change of one parameter can greatly affect the others, making the optimization process more difficult. Additionally, the conventional optimization algorithms, including Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), tend to converge prematurely to local optima. As a result, PEMFC parameter estimation is suboptimal, and PEMFC models are inaccurate and unreliable.

Novel robust optimization techniques are needed to estimate unknown parameters in PEMFC models with sufficient accuracy for model-based optimization to address these challenges. These techniques must be capable of handling the nonlinearities and sensitivities of PEMFCs, compensate for incomplete data, and not prematurely converge in the presence of the complex interdependencies between parameters.

To overcome these challenges, the proposed Fitness Deviation-based Adaptive Differential Evolution (FD-DE) algorithm is used to optimize PEMFC parameters. Several innovative strategies are employed in this algorithm to improve optimization performance. It also includes adaptive parameter control using Gaussian distribution and wavelet basis functions to dynamically adjust the control parameters to improve the exploration of the search space and

Table 5 Evaluation metrics of the FD-DE algorithm applied to FC1

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.6	29	28.9972221	17.4	17.3983333	0.00277787	0.00957886	4.28697E-07
2	2.1	26.31	26.3059404	55.251	55.2424748	0.00405962	0.01542995	9.15584E-07
3	3.58	25.09	25.0935605	89.8222	89.8349465	0.00356047	0.01419078	7.04273E-07
4	5.08	24.25	24.2546268	123.19	123.213504	0.00462678	0.01907949	1.18928E-06
5	7.17	23.37	23.3754238	167.5629	167.601788	0.00542377	0.02320825	1.63429E-06
6	9.55	22.57	22.5846237	215.5435	215.683157	0.01462373	0.06479277	1.18807E-05
7	11.35	22.06	22.0713368	250.381	250.509672	0.01133675	0.05139054	7.14011E-06
8	12.54	21.75	21.7584732	272.745	272.851254	0.0084732	0.03895722	3.98861E-06
9	13.73	21.45	21.4612726	294.5085	294.663273	0.0112726	0.05255293	7.05953E-06
10	15.73	21.09	20.9877521	331.7457	330.13734	0.10224793	0.48481711	0.000580813
11	17.02	20.68	20.6945202	351.9736	352.220734	0.01452023	0.07021388	1.17132E-05
12	19.11	20.22	20.2309972	386.4042	386.614357	0.01099724	0.05438794	6.71885E-06
13	21.2	19.76	19.7709549	418.912	419.144244	0.0109549	0.05543978	6.66721E-06
14	23	19.36	19.3660366	445.28	445.418843	0.00603664	0.03118101	2.0245E-06
15	25.08	18.86	18.8664785	473.0088	473.171281	0.00647852	0.03435059	2.33173E-06
16	27.17	18.27	18.2747331	496.3959	496.524497	0.00473307	0.02590621	1.24455E-06
17	28.06	17.95	17.9533234	503.677	503.770254	0.00332337	0.01851458	6.13598E-07
18	29.26	17.3	17.2928896	506.198	505.989949	0.00711043	0.04110073	2.80879E-06
Average Value of different datasheets						**0.01291984**	**0.06139403**	**3.61043E-05**

to prevent premature convergence. Furthermore, a hybrid trial vector generation strategy based on t-distribution perturbation is proposed to enhance the algorithm's ability to escape local optima and find better solutions. Additionally, the algorithm employs a dimensional replacement mechanism to preserve population diversity to avoid stagnation and to conduct a complete search of the solution space.

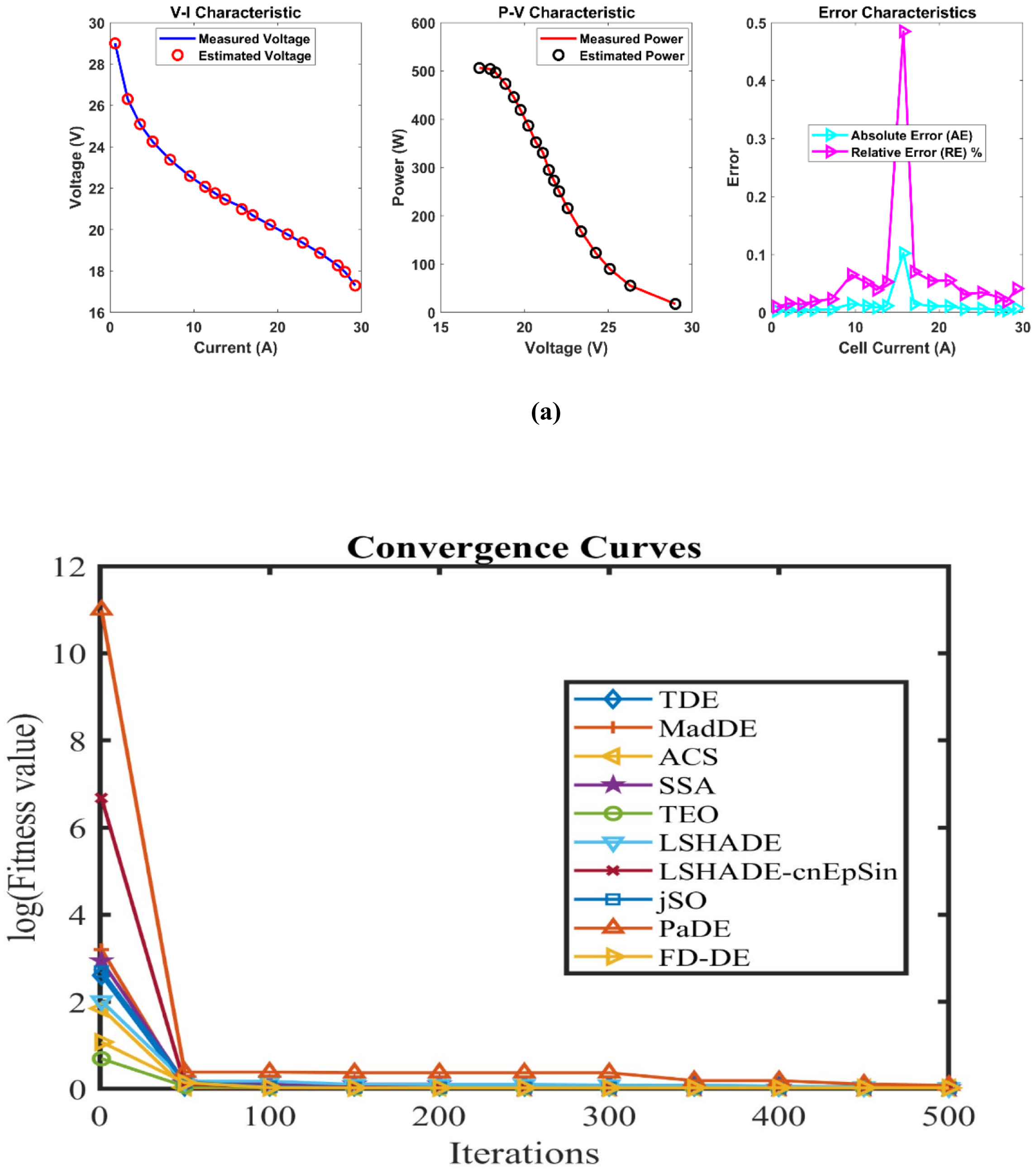
It is an improvement over existing methods because it overcomes the problems that previous optimization techniques have. This improves the exploration and exploitation balance, reduces the sensitivity to control parameter settings, and maintains population diversity effectively for more accurate and reliable parameter estimation of PEMFCs. In several aspects, this approach makes notable contributions. It has adaptive mechanisms and diversity preservation strategies that solve the typical optimization problems such as premature convergence and parameter sensitivity. The algorithm enables more accurate parameter estimation, which is crucial for improving PEMFC model reliability and optimizing fuel cell performance and operational life. The algorithm is shown to perform superior to the other algorithms on twelve different PEMFC case studies under various operating conditions. Compare the FD-DE algorithm to state of the art algorithms and demonstrate that it consistently attains higher accuracy, faster convergence, and greater stability. The FD-DE algorithm is a novel approach to PEMFC parameter optimization problems. The PEMFC model is a robust and reliable tool for researchers and engineers to develop fuel cell modeling and improve its performance, and it successfully handles the nonlinearity and sensitivity of PEMFCs.

The FD-DE algorithm is applied to the PEMFC parameter optimization problem in this paper. It is validated that the FD-DE algorithm is superior to other DE and non-DE variant algorithms in optimizing PEMFC parameters. The analysis followed in this paper are as follows:

1. Implementation of the FD-DE algorithm in a real-world optimization problem related to PEMFC parameter optimization.
2. Designing an optimization model for twelve case studies by operating six commercially available PEMFCs (BCS 500-W [38], STD 250-W stack [39], SR-12 [40], H-12 [40], Nedstack PS6 [39], and HORIZON 500W [41]) under different operating conditions and solving these with FD-DE and other programmed algorithms.
3. Conducting a statistical data analysis to compare FD-DE with other algorithms.
4. Obtaining SSE, Absolute Error (AE), Relative Error (RE), and Mean Bias Error (MBE), as well as I/V and P/V characteristics for different datasheets of practical PEMFC stacks.

The contributions of this analysis are summarized as follows:

- Addressing Limitations of Existing Methods: FD-DE algorithm overcomes the challenges of premature convergence, parameter sensitivity, and inadequate diversity maintenance found in existing optimization methods.
- Innovative Algorithm Design: The integration of adaptive parameter control, hybrid mutation strategies, and diversity preservation mechanisms represents a significant advancement in DE algorithms for complex optimization problems.



(a)

(b)

Fig. 2 FD-DE algorithm analysis for FC1: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

- Enhanced PEMFC Modeling: By achieving more accurate parameter estimation and work contributes to the development of reliable PEMFC models, ultimately aiding in the optimization and control of fuel cell systems.

PEMFC modelling

In this section, firstly, the detailed semi-empirical model and specifications of the selected PEMFC are given. Secondly, the definition of the objective function and statistical

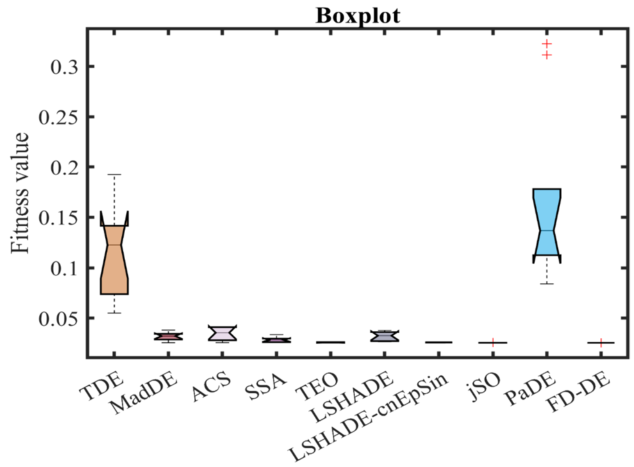


Fig. 2 (continued)

comparison measures such as Mean Biased Error (MBE), objective function efficiency are also discussed.

Semi-empirical electrochemical model

The output voltage of the FC stack (V_{fc}) is obtained using Eq. (1),

$$V_{fc} = (V_{Nernst} - V_{act} - V_{ohmic} - V_{con}) \cdot N_{cell} \tag{1}$$

where V_{act} represents the activation polarization that is caused by the slowness of the reactions at the electrode surface, V_{ohmic} represents the ohmic polarization that is the resistance which implies all electrical and ionic conduction losses through the electrolyte, catalyst layers, cell interconnects, and contacts, V_{con} represents the concentration polarization that is associated with the concentration difference between the fuel/air channel and the chemical species on the electrode surface, and N_{cell} is the number of cells [42]. V_{Nernst} is the reversible cell voltage known as Nernst voltage and can be calculated with Eq. (2) [43, 44].

$$V_{Nernst} =< spanclass =' convertEndash' > 1.229 - 0.85 < /span > \times 10^{-3}(T_{stack} - 298.15) + 4.3085 \times 10^{-5} T_{stack}[ln(p_{H_2}) + 0.5ln(p_{O_2})] \tag{2}$$

where T_{stack} is the stack temperature (K), p_{H_2} is the partial pressure of the hydrogen (bar), and p_{O_2} is the partial pressure of the oxygen (bar). The partial pressure of the hydrogen can be obtained by using Eq. (3) [43].

$$p_{H_2} = 0.5 \cdot RH_a \cdot P^{sat}_{H_2O}[(\exp(\frac{1.635(\frac{I_{fc}}{A_{cell}})}{T^{1.334}_{stack}}) \times \frac{RH_a \cdot P^{sat}_{H_2O}}{P_a})^{-1} - 1] \tag{3}$$

If the pure oxygen is fed to the cathode side of the FC, the partial pressure of oxygen at the cathode can be calculated using Eq. (4).

$$p_{O_2} = P_c - (RH_c \cdot P^{sat}_{H_2O}) \cdot [(\exp(\frac{4.192(\frac{I_{fc}}{A_{cell}})}{T^{1.334}_{stack}}) \cdot \frac{(RH_c \cdot P^{sat}_{H_2O})}{P_c})^{-1} - 1] \tag{4}$$

If the air is used instead of pure oxygen, the partial pressure of oxygen at the cathode can be calculated using Eq. (5).

$$p_{O_2} = P_c - (RH_c \cdot P^{sat}_{H_2O}) - \frac{0.79}{0.21} \cdot p_{O_2} \cdot \exp(\frac{0.291(\frac{I_{fc}}{A_{cell}})}{T^{0.832}_{stack}}) \tag{5}$$

where RH_a and RH_c are relative humidity of vapors in the anode and cathode, respectively. I_{fc} is the FC operating current (A), A_{cell} is the active cell area (cm2), P_a is the anode pressure (bar), and P_c is the cathode pressure (bar). $P^{sat}_{H_2O}$ is the saturation pressure of the water vapor (bar) and can be calculated as a function of the stack temperature using Eq. (6) [43, 44].

$$\log_{10}(P^{sat}_{H_2O}) = 2.95 \times 10^{-2}(T_{stack} - 273.15) - 9.18 \times 10^{-5}(T_{stack} - 273.15)^2 + 1.44 \times 10^{-7}(T_{stack} - 273.15)^3 - 2.18 \tag{6}$$

The activation polarization can be calculated depending on the stack temperature and oxygen concentration with Eq. (7) [43],

$$V_{act} = -[\xi_1 + \xi_2 \cdot T_{stack} + \xi_3 \cdot T_{stack} \ln(C_{O_2}) + \xi_4 \cdot T_{stack} \cdot \ln(I_{FC})] \tag{7}$$

where $\xi_k (k = 1,2,3,4)$ are the semi-empirical coefficients based on theoretical equations with kinetic, thermodynamic, and electrochemical foundations [45], and C_{O_2} is the oxygen concentration (mol · cm − 3) that can be calculated using Eq. (8).

$$C_{O_2} = (\frac{p_{O_2}}{5.08}) \times 10^6 \exp(-\frac{498}{T_{stack}}) \tag{8}$$

The ohmic polarization depends on the membrane resistance, R_m (Ω), and contact resistance, R_C (Ω), as given in Eq. (9).

$$V_{ohmic} = I_{FC} \cdot (R_m + R_C) \tag{9}$$

The membrane resistance depends on the resistivity of the membrane, ρ_m (Ω.cm), membrane thickness, l (cm), and effective membrane area (cm2), which is shown in Eq. (10).

$$R_m = \frac{\rho_m l}{A_{cell}} \tag{10}$$

The membrane resistivity (ρ_m) is calculated by using Eq. (11) for Nafion membranes. (11)

$$\rho_m = \frac{181.6[1 + 0.03(\frac{I_{fc}}{A_{cell}}) + 0.062(\frac{T_{stack}}{303})^2 (J)^{2.5}]}{[\lambda - < spanclass =' convertEndash' > 0.643 - 3 < /span > (\frac{I_{fc}}{A_{cell}})]\exp(4.18(\frac{T_{stack} - 303}{T_{stack}}))}$$

where λ is an adjustable parameter related to the membrane and its preparation process [45]. The concentration polarization is calculated using Eq. (12).

$$V_{con} = -\beta \ln(1 - \frac{J}{J_{max}}) \tag{12}$$

where β is the parametric coefficient (V) that depends on the cell and its operation state, J is the actual current density (A · cm−2), and J_{max} is the maximum current density (A · cm−2).

Fitness function definition

In this study, the model parameters are optimized using different variants of DE and non-DE algorithm to converge the results of the PEMFC model to the results obtained from the literature or manufacturer data to enhance the model. The output voltage is calculated at the points corresponding to each current value using the mathematical equations explained in Section Semi-empirical Electrochemical Model. As a result, the suggested fitness function serves as a metric for the estimated parameters' quality. The SSE, given in Eq. (13) is selected as the fitness function [43].

$$SSE = Min(\sum_{i=1}^{N} [V_{meas}(i) - V_{calc}(i)]^2) \tag{13}$$

where N is the amount of measured data points, i stands for the iteration counter, V_{meas} is the measured voltage of the FC, and V_{calc} denotes the calculated voltage of the FC. Several MADM methods with different principles were also described in Section Ranking of the Algorithms. These methods are used to decide the best-performed MHAs for the H-1000 XP case study. The MBE is calculated by Eq. (14).

$$MBE = \frac{\sum_{i=1}^{N} |V_{meas}(i) - V_{calc}(i)|}{N} \tag{14}$$

The novel FD-DE algorithm

Motivation

When employing numerical methods for optimization, it essential not only to use them but also to understand why a particular set of methods is chosen. In the context of this research, this study focuses on Differential Evolution (DE) algorithms and their variants due to their proven effectiveness in solving complex, nonlinear optimization problems like parameter estimation in Proton Exchange Membrane Fuel Cells (PEMFCs). Traditional DE algorithms, while robust, face limitations such as sensitivity to control parameter settings, premature convergence to local optima, and difficulties in balancing exploration and exploitation during the search process.

To address these limitations, several DE variants have been developed over time, each introducing progressive improvements. The jDE algorithm encoded self-adaptive control parameters in individuals for dynamic adjustment of parameters. But it still had misleading interactions between parameters. In JADE, the scale factor F and crossover rate CR were adapted according to successful trial vectors, but the adaptations of F and CR were dependent on each other and thus may cause parameter interaction problems.

The LSHADE algorithm introduced a historical memory (H-entry pool) to record successful μ_F and μ_{CR} pairs, enhancing robustness in parameter control. It also implemented a linear population size reduction to balance exploration and exploitation. Despite these advancements, LSHADE could be over-tuned for specific problems, potentially limiting its general applicability.

jSO then added inertia weights and adjusted parameter adaptation schemes to further improve performance, and LPalmDE addressed misleading parameter interactions by grouping control parameters and updating them independently. PaDE improved grouping strategies and added a parabolic population size reduction to keep diversity well. Hip-DE used historical population information to incorporate past search experiences in the mutation strategy to guide the current search direction.

Although these progressive improvements have been made, there still exist challenges, such as keeping diversity, avoiding premature convergence, and achieving a better balance between global exploration and local exploitation.

Addressing the limitations of parameter sensitivity, misleading parameter interactions, and diversity loss, the FD-DE algorithm demonstrates superior performance in parameter estimation tasks. It consistently outperforms other state-of-the-art algorithms in terms of accuracy, convergence speed,

Table 6 Comparative performance metrics of optimization algorithms for FC2

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.10593	−0.88598	−0.85320	−0.86027	−0.85973	−1.03379	−1.10849	−1.07545	−1.11055	−0.85461
ξ_2	0.00400	0.00265	0.00327	0.00254	0.00262	0.00360	0.00367	0.00361	0.00376	0.00240
ξ_3	0.000098	4.745E-05	0.000098	4.458E-05	5.045E-05	8.407E-05	7.351E-05	7.621E-05	7.911E-05	3.6E-05
ξ_4	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010
λ	14.40738	14.00002	14.00000	14.00000	14.01421	14.04156	14.00000	14.00791	14.45306	14.00000
R_c	0.00012	0.00013	0.00010	0.00012	0.00013	0.00013	0.00012	0.00011	0.00015	0.00012
B	0.02101	0.01584	0.01959	0.01662	0.01610	0.01577	0.01703	0.01910	0.01572	0.01679
Min.	0.27776	0.27529	0.27590	0.27530	0.27542	0.27579	0.27523	0.27579	0.31769	0.27521
Max.	1.01645	0.30347	0.27606	0.32423	0.29050	0.30000	0.28799	0.28795	0.59080	0.27521
Mean	0.56125	0.28870	0.27601	0.28469	0.28326	0.28759	0.27712	0.27986	0.40397	0.27521
Std.	0.23075404	0.01027219	7.58325E-05	0.01561158	0.0051162	0.00979068	0.00384074	0.00434235	0.09524811	2.54384E-16
RT	8.17505	7.77721	6.98318	7.28058	15.14574	8.03601	7.97731	8.94844	0.15616	15.80297
FR	9	6.1	4	4.6	5.6	6.7	3.8	5	9.2	1

and stability, as evidenced by extensive evaluations on multiple PEMFC case studies. This selection of numerical methods and the development of FD-DE are thus justified by the need to overcome the specific challenges inherent in optimizing complex, nonlinear systems like PEMFCs.

The description of the novel FD-DE algorithm is divided into three sections: the first section outlines the parameter adaptations, the second section details the hybrid trial vector generation strategy, and the third section discusses the diversity adjustment through the replacement of individual dimensions.

Parameter adaptation

The scale factor ***F*** plays a crucial role in generating donor vectors, significantly impacting the overall performance of Differential Evolution (DE). Therefore, adapting the scale factor ***F*** during evolution is a key component of DE research. In the FD-DE algorithm [37], a wavelet basis function-based parameter control for the scale factor ***F*** is proposed for the early stages of evolution, rather than using the commonly employed Cauchy distribution throughout the entire evolution. In this stage, any ***F*** value exceeding 0.6 is truncated to 0.6. This approach is inspired by the wavelet basis function advantage in time and frequency domain localization [46], serving as an alternative for generating the scale factor. The details of generating scale factors are provided in Eq. (15):

$$F_i = \begin{cases} \sqrt{2} \cdot \pi^{-\frac{1}{3}} \cdot (1 - \mu_F^2) \cdot e^{-\frac{\mu_F^2}{2}} + \varepsilon, & if nfe < nfe_{st} \\ randc_i(\mu_F, 0.1), & otherwise \end{cases} \tag{15}$$

where $\varepsilon = 0.1 \cdot sin(\pi \cdot rand_i - 0.8)$. Figure 1 compares the Cauchy function and the Wavelet basis function in generating F values.

The $crossoverrate(CR)$ significantly influences the generation of trial vectors as it determines the probability of parameters in the donor vector being inherited into the trial vector. The FD-DE algorithm employs the Gaussian distribution as used in the literature [47, 48]. Additionally, CR values are restricted to the range [0, 0.6] during the first stage of the evolution and extend to [0, 1] for the remainder of the evolution. The detailed generation of CR is given in Eq. (16):

$$CR_i = \begin{cases} 0, & if\, \mu_{CR} = \emptyset \\ randn_i(\mu_{CR}, 0.1), & otherwise \end{cases} \tag{16}$$

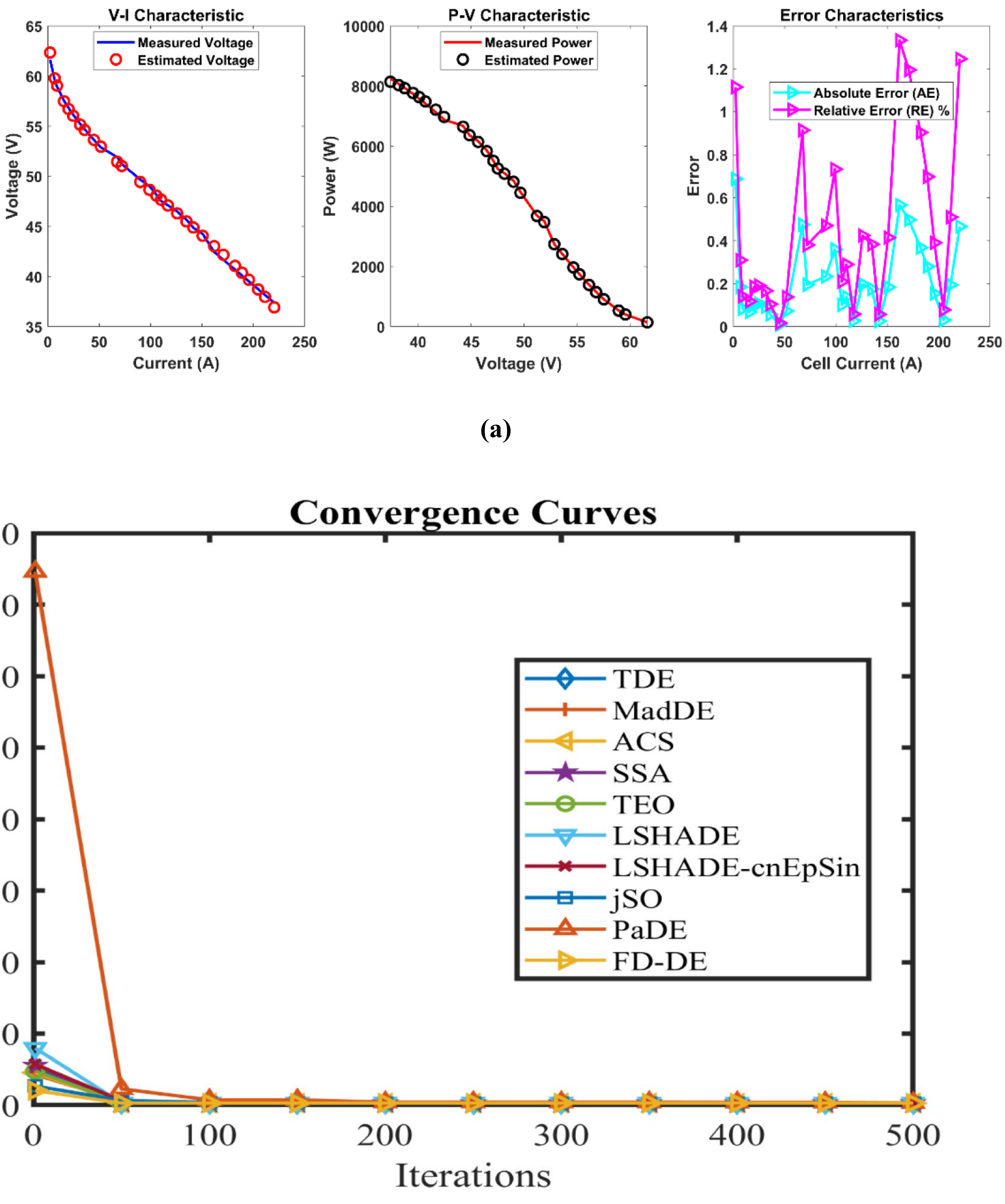
Obviously, both the scale factor (F) and the crossover rate (CR) are truncated to smaller values, no larger than 0.6, in the initial stage of evolution. In the later stages, the ranges of F and CR are extended to (0, 1] and [0, 1] respectively. This strategy helps prevent premature convergence and aids in drawing the population out of local optima.

Table 7 Evaluation metrics of the FD-DE algorithm applied to FC2

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	2.25	61.64	62.327094	138.69	140.23596	0.6870937	1.114688	0.0162792
2	6.75	59.57	59.753916	402.0975	403.33893	0.183916	0.3087393	0.0011664
3	9	58.94	59.023005	530.46	531.20705	0.0830053	0.1408302	0.0002376
4	15.75	57.54	57.472458	906.255	905.19121	0.0675421	0.1173828	0.0001573
5	20.25	56.8	56.695017	1150.2	1148.0741	0.1049832	0.1848296	0.0003801
6	24.75	56.13	56.023048	1389.2175	1386.5704	0.1069518	0.190543	0.0003944
7	31.5	55.23	55.138044	1739.745	1736.8484	0.0919559	0.1664963	0.0002916
8	36	54.66	54.603004	1967.76	1965.7081	0.0569962	0.104274	0.000112
9	45	53.61	53.618874	2412.45	2412.8493	0.0088744	0.0165537	2.716E-06
10	51.75	52.86	52.932655	2735.505	2739.2649	0.0726547	0.1374473	0.000182
11	67.5	51.91	51.435598	3503.925	3471.9028	0.4744023	0.9138939	0.0077606
12	72	51.22	51.025405	3687.84	3673.8292	0.1945948	0.3799195	0.0013058
13	90	49.66	49.426729	4469.4	4448.4056	0.2332711	0.4697364	0.0018764
14	99	49	48.641019	4851	4815.4609	0.3589812	0.7326148	0.0044437
15	105.8	48.15	48.049175	5094.27	5083.6028	0.1008246	0.209397	0.0003505
16	110.3	47.52	47.657409	5241.456	5256.6122	0.1374087	0.2891597	0.0006511
17	117	47.1	47.072842	5510.7	5507.5225	0.027158	0.0576602	2.543E-05
18	126	46.48	46.28307	5856.48	5831.6668	0.19693	0.4236877	0.0013373
19	135	45.66	45.485316	6164.1	6140.5177	0.1746836	0.3825746	0.0010522
20	141.8	44.85	44.875522	6359.73	6363.349	0.0255219	0.056905	2.246E-05
21	150.8	44.24	44.056856	6671.392	6643.7739	0.183144	0.4139783	0.0011566
22	162	42.45	43.015705	6876.9	6968.5442	0.5657049	1.3326382	0.0110352
23	171	41.66	42.157523	7123.86	7208.9364	0.4975231	1.1942465	0.0085355
24	182.3	40.68	41.04752	7415.964	7482.9629	0.3675198	0.903441	0.0046576
25	189	40.09	40.369551	7577.01	7629.8452	0.2795514	0.6973096	0.0026948
26	195.8	39.51	39.664141	7736.058	7766.2388	0.1541412	0.390132	0.0008193
27	204.8	38.73	38.699846	7931.904	7925.7285	0.0301536	0.077856	3.135E-05
28	211.5	38.15	37.955786	8068.725	8027.6488	0.1942139	0.5090796	0.0013007
29	220.5	37.38	36.914224	8242.29	8139.5864	0.4657762	1.2460571	0.0074809
Average Value of different datasheets						**0.2112234**	**0.4538645**	**0.0026118**

Figure 1 illustrates the differences between two methods used for generating scale factor (F) values in the FD-DE algorithm: the Cauchy distribution and the Wavelet basis function. Traditionally, DE algorithms employ a Cauchy distribution to generate the scale factor F throughout the entire evolution. This method can sometimes lead to larger, unstable F values early in the evolution, which may negatively affect convergence speed. In the FD-DE algorithm, propose a wavelet basis function-based control mechanism for the early stages of evolution. This method localizes F in both the time and frequency domains, thus providing smoother and more stable F values. As shown in the figure, F values remain more controlled and do not exceed 0.6 during early evolution stages, in contrast to the Cauchy-based approach. By comparing the two methods, the figure highlights how the wavelet-based control introduces stability and adaptability in generating F values, particularly at the early stages of the algorithm run, contributing to better convergence performance. This enhanced explanation should provide clarity on the purpose and significance of Fig. 1 in the context of the FD-DE algorithm.

The $H-entry$ pool recording μ_F and μ_{CR} pairs, as proposed in LSHADE [5], is also retained in the FD-DE algorithm to enhance the robustness of parameter control. Only one entry is updated in each generation, with the entry pool being updated in a circular manner from the first to the last entry, and then back to the first. The concept of using good control parameters to update their own distributions, as seen in JADE, is further refined in the FD-DE algorithm, with details provided in Eqs. (17) and (18) respectively:



(a)

(b)

Fig. 3 FD-DE algorithm analysis for FC2: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

$$\begin{cases} w_k = \dfrac{d_{f_i}}{\sum_{k\in S} d_{f_i}} \\ mean_{WL}(S_F) = \dfrac{\sum_{k=1}^{|S_F|} w_k \cdot S_F^2(k)}{\sum_{k=1}^{|S_F|} w_k \cdot S_F(k)} \\ \mu_{F,k,G+1} = \begin{cases} \frac{(mean_{WL}(S_F)+\mu_{F,k,G})}{2}, & if S_F \neq \varnothing \\ \mu_{F,k,G}, & otherwise \end{cases} \end{cases} \tag{17}$$

$$\begin{cases} w_k = \dfrac{d_{f_i}}{\sum_{k\in S} d_{f_i}} \\ mean_{WL}(S_{CR}) = \dfrac{\sum_{k=1}^{|S_{CR}|} w_k \cdot S_{CR}^2(k)}{\sum_{k=1}^{|S_{CR}|} w_k \cdot S_{CR}(k)} \\ \mu_{CR,k,G+1} = \begin{cases} mean_{WL}(S_{CR}), & if S_{CR} \neq \varnothing \\ \mu_{CR,k,G}, & otherwise \end{cases} \end{cases} \tag{18}$$

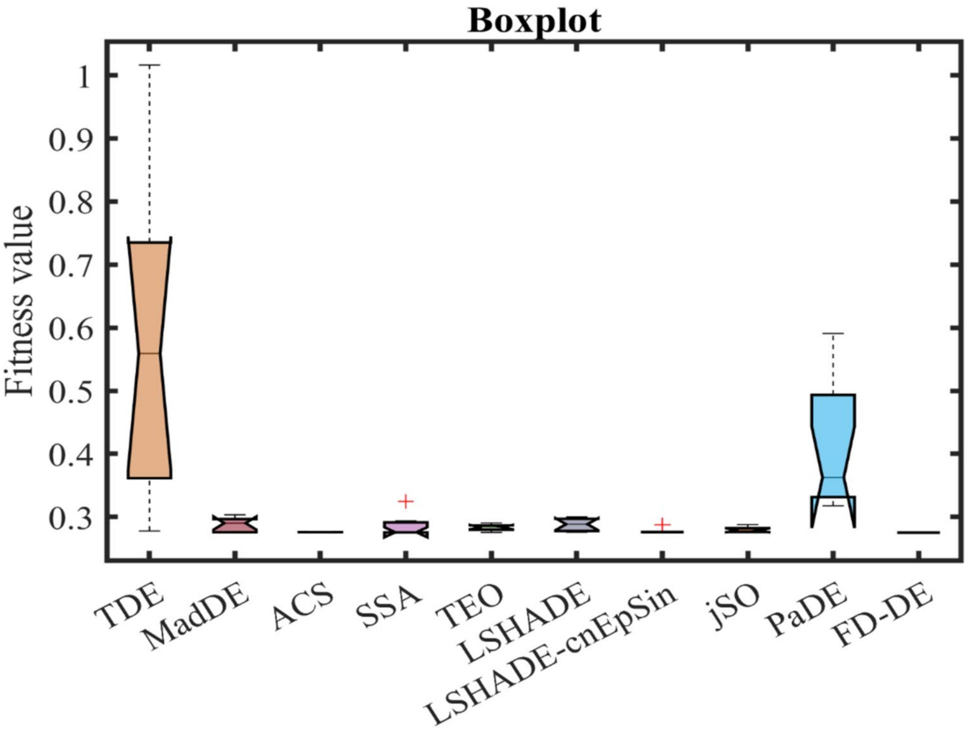


Fig. 3 (continued)

where d_{f_i} represents the fitness deviation of the ith individual in the population, calculated using Eq. (19):

$$\begin{cases} d_{f_i} = \dfrac{|\Delta f_i - \overline{\Delta f}|}{\Delta f_i} \\ \Delta f_i = f(X_{i,G}) - f(U_{i,G}) \\ \overline{\Delta f} = \dfrac{\sum_{i=1}^{PS}(f(X_{i,G}) - f(U_{i,G}))}{PS} \end{cases} \tag{19}$$

Here, $\overline{\Delta f}$ represents the mean value of the fitness differences across all individuals in the population, while Δf_i denotes the fitness improvement of the i $^{\text{th}}$ individual. The update of control parameters μ_F and μ_{CR} is performed only if the success set SSS of individuals is not empty, meaning Δf_i satisfies $\Delta f_i > 0$.

In the early stages of evolution, a parameter control based on wavelet basis function is used to control the scale factor F. As compared to the traditional functions, the wavelet basis function provides superior localization properties in both the time and frequency domains which enables more localized adaptation of F. A wavelet basis function is chosen for F adaptation because it enables a controlled exploration of the search space. The wavelet function localization property assures that the scale factor adapts dynamically according to the landscape of the search space, making the algorithm better escape local optima in the early stage of evolution. The FD-DE algorithm balances exploration and exploitation dynamically by adapting the parameters F and CR to the wavelet basis and Gaussian functions, respectively, in the course of the evolution. The truncation thresholds and adaptation mechanisms are based on empirical observations and are intended to speed up convergence without premature convergence.

Trial vector generation strategy

While the “$DE/target-to-pbest/1/bin$” mutation strategy with an external archive [48] has shown excellent performance in recent competitions, it still struggles with premature convergence and local optima when dealing with complex optimization problems. To address this, a t-distribution-based perturbation strategy is introduced as a supplement to the “$DE/target-to-pbest/1/bin$” mutation strategy. The idea is that the standard deviation of the global best can serve as the perturbation range for the target vector.

The main mutation strategy in FD-DE algorithm is defined in Eq. (20):

$$V_{i,G} = X_{i,G} + F \cdot (X^{p}_{best,G} - X_{i,G}) + F \cdot (X_{r_1,G} - \tilde{X}_{r_2,G}). \tag{20}$$

The trial vector $U_{i,G}$ is then calculated using a crossover operation based on a specified crossover rate (CR). Following this, the t-distribution-based perturbation is applied to the trial vector with a cetain probability $\boldsymbol{\tau} = \tau = 0.05$. The perturbation details are provided in Eq. (21):

$$U_{j,i,G} = X_{j,i,G} + rand_{j,i,G} \cdot std(X_{gbest,G}) \cdot (tpdf(G, 1) + 1) \tag{21}$$

where $U_{j,i,G}$ and $X_{j,i,G}$ denote the j th parameter of the trial vector $U_{i,G}$ and the target vector $X_{i,G}$ respectively, $std(X_{gbest,G})$ denotes the standard deviation of $X_{gbest,G}$, and $tpdf(G, 1)$ denotes the probability density function of -distribution. The pseudo code of the trial vector generation strategy is given in Algorithm 1. Rather than the normal distribution t-distribution is used since it has fatter tails, leading to larger probability values for those observations away from the mean. Its property of smoothly increasing the chance to generate significant perturbations improves the chances of the algorithm to escape from local optima and visit hitherto unexplored areas of the search space. Selective perturbation is applied to balance between exploration and exploitation. However, the small probability τ of the perturbation guarantees that during the convergence process do not inject too much diversity, but is sufficient to inject diversity when needed. The perturbation scale is adapted to the current state of the population by using the standard deviation of the global best individual, enabling context sensitive adjustments.

Algorithm 1 Trial vector generation strategy

```
for $i = 1$ to $PS$ do
    Calculate $V_{i,G}$ according to Eq.(20);
    Generate $j_{\mathrm{rand}}$ = randint $(1, D)$;
    for $j = 1$ to $D$ do
        if $j = j_{\mathrm{rand}}$ or $rand(0,1) < CRi$ then
            $U_{j,i,G} = V_{j,i,G}$;
        else
            if $rand > \tau$ then
                $U_{j,i,G} = X_{j,i,G}$
            else
                Perturb $U_{i,G}$ according to Eq.(21);
            end if
        end if
    end for
end for
```

Dimensional replacement of individuals

Beyond perturbation in the trial vector, a diversity indicator for the population is introduced, with dimensional replacement of individuals initiated if necessary to help the population escape local optima. To measure individual diversity, a surrogate hyper-volume model to compute the geographic distribution of individuals within the population. This model enhances a previous version proposed in [6]. The hyper-volume of the individuals can be calculated using Eq. (22):

$$V_{hp} = \sqrt{\prod_{i=1}^{D} |u_i - l_i|} \tag{22}$$

where u_i and l_i represent the upper and lower bounds of the ith dimension of the search domain. The diversity indicator d_{in} din can then be calculated using Eq. (23):

$$\begin{cases} d_{in} = \sqrt{V_{pop}/V_{hp}} \\ V_{pop} = \sqrt{\prod_{i=1}^{D} |(u_{x_i} + l_{x_i})/2|} \end{cases} \tag{23}$$

where V_{pop} denotes the center cube of the population, and u_{x_i} and l_{x_i} represent the upper and lower bounds of the *ith* dimension of the population.

In addition to the diversity indicator, a label tracking the improvement status of each individual is used in the dimensional replacement mechanism. The sum of the labels recording individuals without performance improvement is calculated according to Algorithm 2. When the algorithm enters a state where $d_{in} < \xi$ ($\xi = 0.01$) and $C_n > k \cdot PS \cdot D$ ($k = 0.6$), dimensional replacement of individuals is carried out as per Eq. (24):

$$X_{i,G}(j) = \begin{cases} X_{rand_i,G}(j), & ifj \in R \\ X_{i,G}, & otherwise \end{cases} \tag{24}$$

Algorithm 2 Calculate counter and C$_n$

```
$C_n = 0$
for $i = 1$ to $PS$ do
    if $f(X_{i,G}) > f(U_{i,G})$ then
        $counter\ (i) = counter(i) + 1$
        $Cn = Cn + counter(i)$
    else
        $counter\ (i) = 0;$
    end if
end for
```

Algorithm 3 Pseudo code of the FD-DE algorithms

```
Require: Dimension $D$ , bound constraints $[L, U]^D$, maximum
    number of function evaluations $nfesmax$, objective $f(X)$
Ensure: Global best value $f(X_{gbest})$, global best individual $X_{gbest}$,
    number of function evaluations $nfes$;
    Initialize the population $P == \{X_1, X_2, \ldots, X_{PS}\}$,
    calculate the fitness values and label the global best individual;
    Initialize the parameters and the external archive $A$;
    while $nfes < nfes_{max}$ do
        Calculate the control parameters and the trial vectors;
        for $i = 1$ to $PS$ do
            Update the population;
            if $f(U_{i,G} < f(X_{i,G})$ then
                $X_{i,G} \rightarrow A$
                $CR_{i,G} \rightarrow S_{CR}, F_{i,G}$;
            end if
        end for
        Update $A, \mu_{CR}$ and $\mu_F$;
        for $i = 1$ to $PS$ do
            if $C_n > n \,\&\, d_{in} < 0.01$ then
                Adjust the ith individual according to Eq. (24)
                Calculate the fitness value $f(Xi, G)$;
                $nfes = nfes + 1$;
            end if
        end for
        Calculate $PS_G + 1$ according to $PS_{G+1} = round[\frac{PS_{min} - PS_{max}}{nfe_{max}} \cdot nfe + PS_{max}]$
        if $PS_G < PS_{G+1}$ then
            Individuals are sorted depending on their fitness;
                The worse $PS_G - PS_{G+1}$ individuals are removed from the population;
            end if
            $G = G + 1$
    end while
```

Table 8 Comparative performance metrics of optimization algorithms for FC3

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.05811	−0.85321	−0.85320	−1.04157	−1.02016	−0.91697	−1.10508	−1.04421	−0.85320	−0.85320
ξ_2	0.00385	0.00259	0.00229	0.00311	0.00372	0.00334	0.00380	0.00346	0.00249	0.00256
ξ_3	9.689E-05	5.574E-05	0.000036	5.149E-05	9.477E-05	9.11E-05	8.303E-05	7.327E-05	4.883E-05	5.326E-05
ξ_4	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010
λ	22.03196	22.99997	23.00000	21.49034	22.89824	20.71398	22.17546	22.82581	14.00000	23.00000
R_c	0.00010	0.00070	0.00067	0.00076	0.00068	0.00070	0.00065	0.00066	0.00033	0.00067
B	0.18519	0.17473	0.17532	0.17297	0.17514	0.17393	0.17541	0.17549	0.17591	0.17532
Min.	0.26028	0.24231	0.24228	0.24270	0.24230	0.24251	0.24233	0.24230	0.24793	0.24228
Max.	0.84251	0.24637	0.24746	0.24572	0.24289	0.24618	0.24264	0.24254	1.10674	0.24293
Mean	0.36385	0.24352	0.24433	0.24426	0.24250	0.24432	0.24247	0.24236	0.45305	0.24248
Std.	0.19060022	0.00122343	0.00199618	0.00106817	0.00016706	0.00149826	0.00010584	7.60948E-05	0.2475579	0.00031068
RT	6.04349	5.68226	4.88308	5.12815	10.92912	5.81966	5.73288	6.65848	0.11180	11.50124
FR	9.3	5.3	5.8	6.6	3.3	6.7	3.4	2.3	9.7	2.6

Set ξ = 0.01 to ensure that the replacement mechanism is activated before the population converges too early. The value is chosen empirically so that the diversity is maintained without too much disturbance to the convergence process. The dimensional replacement mechanism effectively reintroduces variability into some dimensions of the individuals, allowing the population to escape local optima and explore new regions of the search space. The algorithm preserves useful genetic material by replacing only a subset of dimensions, and yet still enhances diversity.

This means that the *jth* dimension of the ith individual is replaced by the *jth* dimensional parameter of a randomly selected individual. The complete pseudo code for FD – DE algorithm is provided in Algorithm 3. It is evident that the core structure of this pseudo code is quite similar to that of various DE algorithms such as jSO, PaDE, LPalmDE, and Hip-DE. The primary distinctions among these algorithms are found in the generation of trial vectors, parameter control, and the diversity enhancement technique [37]. The parameter choices in the FD-DE algorithm are guided by both theoretical considerations and empirical observations:

- **Probability τ for Perturbation:** Set to 0.05, this small value ensures that the t-distribution-based perturbation is applied infrequently, preventing excessive randomness while still providing opportunities to escape local optima.
- **Diversity Threshold ξ:** A value of 0.01 is selected to detect critical levels of diversity loss. This threshold is low enough to avoid premature triggering of the replacement mechanism but sensitive enough to act before convergence stalls.
- **Proportion k for Dimensional Replacement:** The value k = 0.6 is chosen to initiate replacement when a significant portion of the population shows stagnation. This proportion balances the need for diversity with the preservation of valuable genetic information.
- **Truncation of F and CR:** Limiting F and CR to certain ranges during early evolution stages prevents extreme parameter values that could lead to unstable behavior or slow convergence.

These parameter settings have been validated through extensive experimentation and are intended to provide a robust performance across a wide range of optimization problems.

Result analysis

The simulation tests have been carried out to validate the application of FD-DE algorithm for single objective optimization problem related with estimating the parameters

Table 9 Evaluation metrics of the FD-DE algorithm applied to FC3

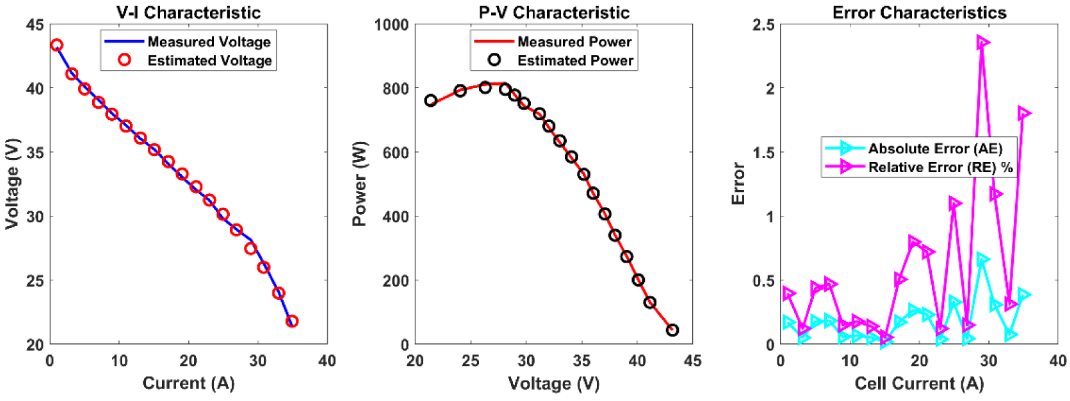
S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	1.004	43.17	43.340798	43.34268	43.514161	0.170798	0.3956405	0.0016207
2	3.166	41.14	41.090066	130.24924	130.09115	0.0499336	0.1213748	0.0001385
3	5.019	40.09	39.914501	201.21171	200.33088	0.1754993	0.4377632	0.0017111
4	7.027	39.04	38.857141	274.33408	273.04913	0.1828592	0.4683895	0.0018576
5	8.958	37.99	37.933453	340.31442	339.80787	0.0565467	0.1488463	0.0001776
6	10.97	37.08	37.014525	406.7676	406.04934	0.0654747	0.1765769	0.0002382
7	13.05	36.03	36.079894	470.1915	470.84262	0.0498941	0.1384792	0.0001383
8	15.06	35.19	35.171352	529.9614	529.68057	0.0186476	0.0529912	1.932E-05
9	17.07	34.07	34.242077	581.5749	584.51225	0.1720767	0.5050681	0.001645
10	19.07	33.02	33.283114	629.6914	634.70899	0.2631144	0.7968333	0.0038461
11	21.08	32.04	32.270689	675.4032	680.26611	0.2306885	0.7200016	0.0029565
12	23.01	31.2	31.237682	717.912	718.77906	0.0376819	0.1207754	7.888E-05
13	24.94	29.8	30.12736	743.212	751.37635	0.3273597	1.0985226	0.0059536
14	26.87	28.96	28.917122	778.1552	777.00307	0.0428779	0.148059	0.0001021
15	28.96	28.12	27.457745	814.3552	795.17629	0.6622551	2.3551035	0.0243657
16	30.81	26.3	25.991793	810.303	800.80713	0.3082074	1.1718913	0.0052773
17	32.97	24.06	23.984857	793.2582	790.78073	0.0751431	0.3123153	0.0003137
18	34.9	21.4	21.785622	746.86	760.3182	0.3856218	1.8019712	0.0082613
Average Value of different datasheets						**0.1819267**	**0.6094779**	**0.0032612**

of PEM fuel cells. The FD-DE algorithm as well as other DE variants namely, TDE [49], MadDE [50], LSHADE [5], LSHADE-cnEpSin [51], jSO [52], PaDE [53] and non-DE variants like, ACS [54], SSA [55], and TEO [56], have been tested for estimating the parameters of three different modules of fuel cells, namely BCS 500-W, STD 250-W stack, Nedstack PS6, SR-12PEM, H-12 and HORIZON 500W PWM under various operating conditions depicted in Table 2. The data sheet parameters of these commercial PEMFC stacks are obtained from [8, 11, 57–59]. Moreover, the estimated model parameters are $\xi_1, \xi_2, \xi_3, \xi_4, \lambda, R_c$ and B in PEMFC. The upper and lower limits of the unknown parameters for all case studies are given in [11, 60, 61]. Defaults setting of mentioned algorithms used in this paper is shown in Table 3.

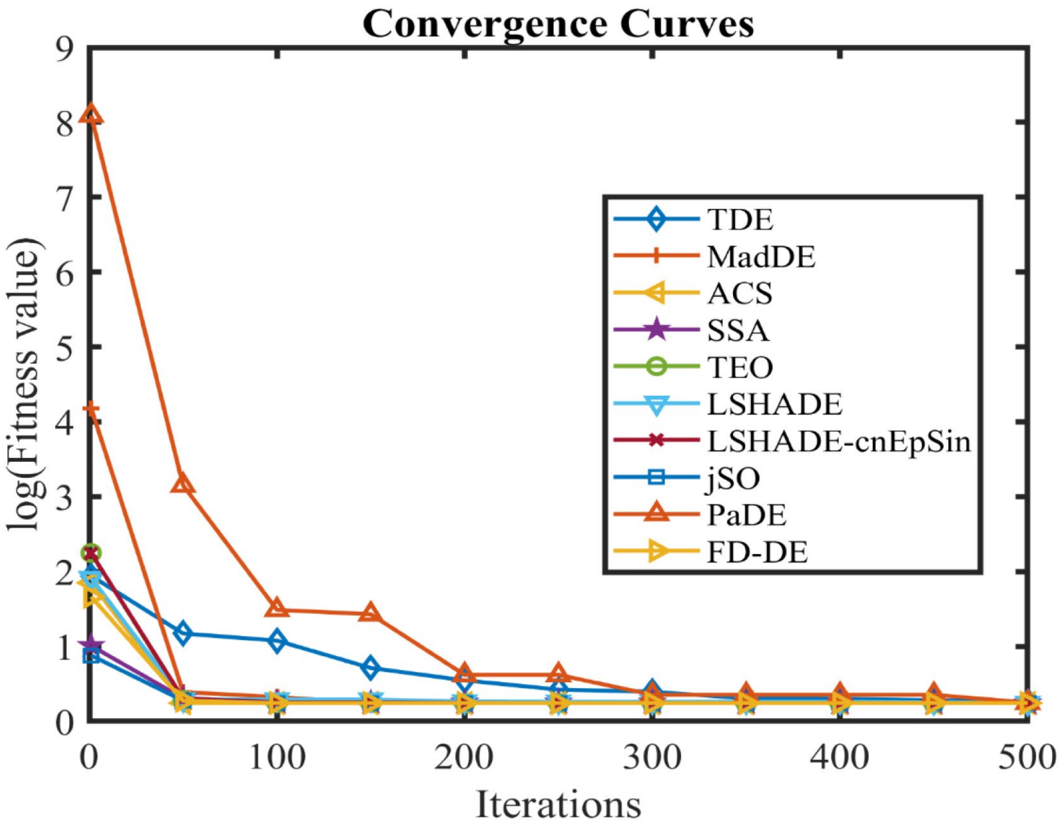
In this study, a diverse set of numerical methods, including PaDE, FD-DE, TDE, MadDE, ACS, SSA, TEO, LSHADE, LSHADE-cnEpSin, and jSO, were chosen for their strengths in handling complex, non-linear, and multi-objective optimization problems. These methods, particularly the differential evolution (DE)-based algorithms like PaDE, FD-DE, and LSHADE variants, are well-known for their global search capabilities and their ability to balance exploration and exploitation. PaDE and FD-DE, for example, dynamically adapt control parameters during the search process, ensuring robust convergence toward global optima while maintaining computational efficiency. The bio-inspired algorithms like ACS and SSA were included for their ability to diversify the search process, which prevents premature convergence and helps explore various regions of the solution space effectively. The population-based nature of these algorithms, combined with adaptive strategies, ensures they can handle large-scale, non-convex, and constrained optimization tasks with consistent performance across different cases.

Moreover, the optimized parameters using FD-DE method has been used to estimate the performance and characteristics of the PEMFC at different operating conditions. Furthermore, the characteristics have been compared with the measured data of each module. All cases are analysed with maximum iterations 500, number of run 30 and population size 40 on Matlab 2021a of a PC with Windows Server 2019 operating system CPU i7-11700 k@3.6 GHz.

The parameter settings for each algorithm were derived from the literature and, where applicable, adjusted to reflect the problem dimensions and characteristics. For example, algorithms like LSHADE and jSO were configured based on dynamic population size and control parameters from reputable sources (cited in the paper). By using these established setting ensured that each algorithm performed optimally in accordance with how they have been originally designed and tested. For non-DE



(a)



(b)

Fig. 4 FD-DE algorithm analysis for FC3: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

algorithms, similar attention was given to parameters like pheromone factors for ACS or random control coefficients for SSA.

The performance of optimization algorithms, particularly metaheuristic algorithms like Differential Evolution (DE) and its variants, is significantly influenced by the choice of hyperparameters such as population size (PS), maximum iterations, scale factor (F), and crossover rate (CR). A careful selection and justification of these parameters are crucial for ensuring

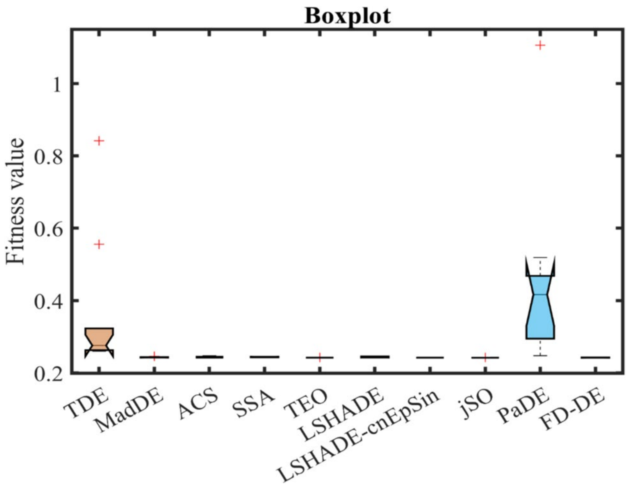


Fig. 4 (continued)

the effectiveness and efficiency of the algorithm. This section provides a comprehensive sensitivity analysis of key hyperparameters for the FD-DE algorithm, demonstrating their impact on algorithm performance and justifying the chosen settings.

Hyperparameter settings used in the experiments

In the experiments, the following default parameter settings for the FD-DE algorithm were employed as shown in Table 3. The initial hyperparameter settings were chosen based on the following considerations:

1. **Population size (PS):** The population size in DE algorithms affects the diversity of the search and the convergence speed. A population size proportional to the problem dimensionality (D) ensures sufficient diversity while keeping computational cost manageable. The formula $PS = 25 \times \ln(D) \times \sqrt{D}$ has been found to provide a good balance between exploration and exploitation for a wide range of problem sizes [52].
2. **Maximum iterations:** A maximum iteration count of 500 was selected to allow the algorithm sufficient opportunity to explore the search space and converge to an optimal or near-optimal solution while keeping the computational time reasonable.
3. **Scale factor (F) and crossover rate (CR):** The adaptive control of F and CR is a key feature of the FD-DE algorithm. Initial values of $F = 0.5$ and $CR = 0.8$ are commonly used starting points in DE algorithms and have been effective in previous studies [5, 37]. The adaptive mechanisms adjust these values during the evolution process based on the algorithm performance.

Table 10 Comparative performance metrics of optimization algorithms for FC4

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−0.96199	−0.86822	−1.19969	−0.92241	−0.85711	−1.15326	−1.07991	−0.91018	−1.00290	−0.90216
ξ_2	0.00204	0.00164	0.00258	0.00178	0.00185	0.00315	0.00245	0.00204	0.00236	0.00252
ξ_3	5.017E-05	4.214E-05	0.000036	4.036E-05	5.953E-05	8.725E-05	5.29E-05	6.139E-05	6.385E-05	9.798E-05
ξ_4	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011	−0.00011
λ	14.00000	14.00000	14.00000	14.03344	14.00026	14.00000	14.00000	14.00032	21.57316	14.00000
R_c	0.00080	0.00080	0.00080	0.00042	0.00080	0.00079	0.00080	0.00080	0.00068	0.00080
B	0.01360	0.01360	0.01360	0.01360	0.01364	0.01369	0.01360	0.01360	0.01399	0.01360
Min.	0.10291	0.10291	0.10291	0.10332	0.10292	0.10295	0.10291	0.10292	0.10477	0.10291
Max.	0.10869	0.10364	0.10443	0.10416	0.10318	0.10369	0.10292	0.10292	0.13638	0.10364
Mean	0.10536	0.10317	0.10336	0.10364	0.10301	0.10339	0.10292	0.10292	0.11182	0.10299
Std.	0.00230438	0.0003378	0.00063478	0.00022409	7.87165E-05	0.00029752	9.35334E-07	9.85523E-07	0.0094773	0.00022959
RT	6.00774	5.67768	4.86904	5.18958	11.02792	5.84148	5.74661	6.80611	0.10690	11.51400
FR	6.7	4.5	4.45	7.4	5.5	7.1	3.3	4.4	9.8	1.85

Table 11 Evaluation metrics of the FD-DE algorithm applied to FC4

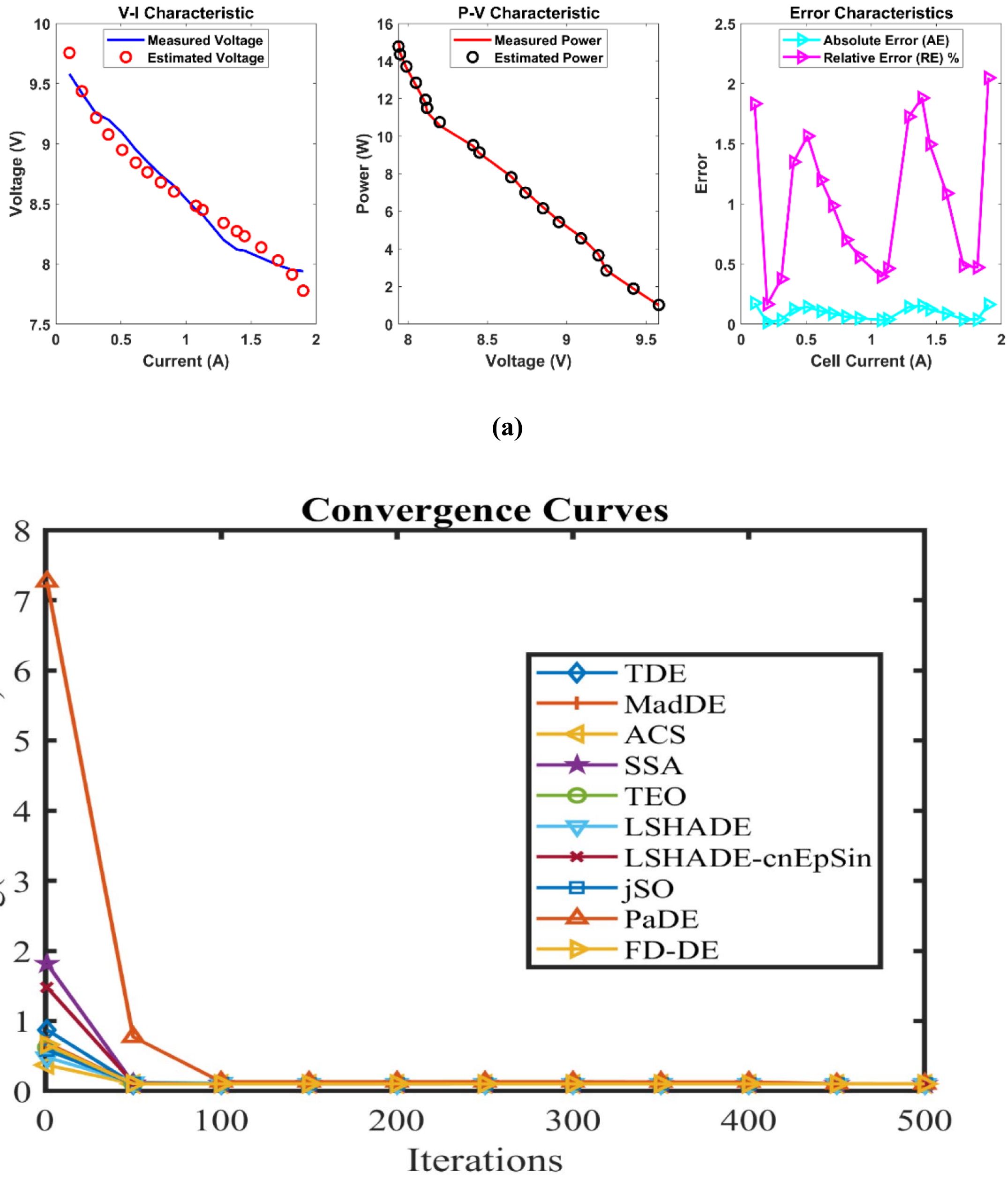
S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.104	9.58	9.7555309	0.99632	1.0145752	0.1755309	1.8322639	0.0017117
2	0.2	9.42	9.4355341	1.884	1.8871068	0.0155341	0.1649054	1.341E-05
3	0.309	9.25	9.2153061	2.85825	2.8475296	0.0346939	0.3750695	6.687E-05
4	0.403	9.2	9.0759953	3.7076	3.6576261	0.1240047	1.3478768	0.0008543
5	0.51	9.09	8.947893	4.6359	4.5634254	0.142107	1.5633333	0.0011219
6	0.614	8.95	8.8427151	5.4953	5.4294271	0.1072849	1.1987139	0.0006394
7	0.703	8.85	8.7628619	6.22155	6.1602919	0.0871381	0.9846108	0.0004218
8	0.806	8.74	8.6786862	7.04444	6.9950211	0.0613138	0.7015309	0.0002089
9	0.908	8.65	8.6015883	7.8542	7.8102422	0.0484117	0.5596729	0.0001302
10	1.076	8.45	8.4833946	9.0922	9.1281325	0.0333946	0.3952019	6.196E-05
11	1.127	8.41	8.4488683	9.47807	9.5218746	0.0388683	0.4621681	8.393E-05
12	1.288	8.2	8.341385	10.5616	10.743704	0.141385	1.7242077	0.0011105
13	1.39	8.12	8.2726638	11.2868	11.499003	0.1526638	1.8800956	0.0012948
14	1.45	8.11	8.2311997	11.7595	11.93524	0.1211997	1.4944475	0.0008161
15	1.578	8.05	8.1375159	12.7029	12.841	0.0875159	1.0871542	0.0004255
16	1.707	7.99	8.0288572	13.63893	13.705259	0.0388572	0.4863226	8.388E-05
17	1.815	7.95	7.9126038	14.42925	14.361376	0.0373962	0.4703925	7.769E-05
18	1.9	7.94	7.7774144	15.086	14.777087	0.1625856	2.0476778	0.0014686
Average Value of different datasheets						**0.0894381**	**1.0430914**	**0.0005884**

PEMFC FC1

In Table 4 analysis, FD-DE stands out for its remarkable consistency and precision among the evaluated algorithms. It achieves the lowest variability in its performance metrics with a minimum and maximum value close at 0.02549 and 0.0255, respectively, and an extraordinarily low standard deviation of 2.45E-06. This underscores FD-DE reliability and precision in reaching near-optimal solutions consistently across different runs. FD-DE also shows the highest stability but has a comparatively longer runtime of 11.74502 s, reflecting a trade-off between runtime efficiency and output stability. Despite its longer runtime, FD-DE secures the best Friedman Rank (FR) of 1, indicating its superior performance over other algorithms. Notably, jSO also performs exceptionally well with the second-best FR of 2.1 and minimal standard deviation, positioning it as another reliable option, especially when considering its shorter runtime of 6.97512 s compared to FD-DE. In Table 4 and Table 5 along with Fig. 2, highlight the exceptional performance of FD-DE. The box-plot for Fig. 2(c), may not visually emphasize minor differences between algorithms with small variance (such as ACS and SSA), the plot effectively captures both the consistency and variability of the results.

PEMFC FC2

In the evaluation of various differential evolution and optimization algorithms, FD-DE demonstrates significant superiority across multiple metrics when compared to other algorithms such as TDE, MadDE, ACS, SSA, TEO, LSHADE, LSHADE-cnEpSin, jSO, and PaDE. In Table 6, FD-DE maintains the lowest variability in results with a standard deviation nearly zero (2.54E-16), indicating exceptionally stable performance across runs. It also boasts the best minimum and maximum performance scores (0.27521 for both), underscoring its consistency in achieving the lower bound of its performance spectrum, which none of the other algorithms approach. Notably, its mean performance, although not the lowest at 0.27521, complements its minimal spread, reinforcing its reliability. The RT (Run Time) for FD-DE is notably higher at 15.80297 s, suggesting a trade-off between



(a)

(b)

Fig. 5 FD-DE algorithm analysis for FC4: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

stability and computational speed, but this is balanced by its superior Friedman Rank (FR) of 1, highlighting it as the top performer overall. Table 6 and Table 7, along with Fig. 3, highlight the exceptional performance of FD-DE.

PEMFC FC3

In Table 8 comprehensive comparison of various optimization algorithms, FD-DE exhibits remarkable precision and stability, as evidenced by its consistently low standard

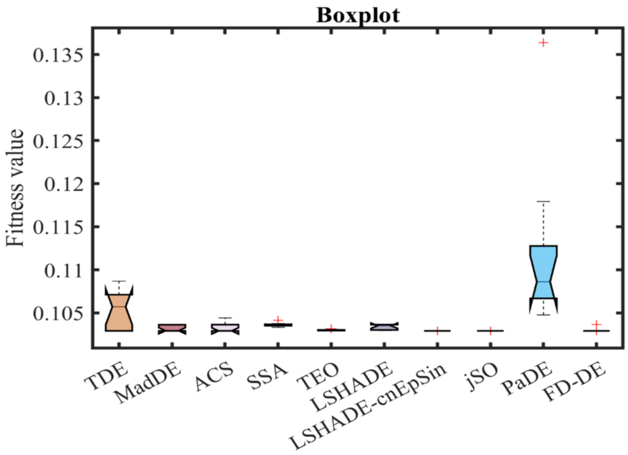


Fig. 5 (continued)

deviation (0.00031068) and the smallest range in minimum and maximum values (0.24228 to 0.24293), indicating minimal fluctuation in performance. The FD-DE algorithm not only excels in consistency but also in efficiency with a Runtime (RT) of 11.50124, which is higher compared to others, suggesting a trade-off between thoroughness of search and computational time. This is further supported by its outstanding Friedman Rank (FR) of 2.6, which is significantly lower than most other algorithms, such as TDE and jSO, indicating superior ranking in terms of optimization performance. The FD-DE mean performance at 0.24248 is notably better than most other competitors like LSHADE-cnEpSin and jSO, which display mean values of 0.24247 and 0.24236 respectively, thus highlighting FD-DE capability to achieve close to the best possible optimization outcomes more consistently than others. In Table 8 and Table 9 along with Fig. 4 highlight the exceptional performance of FD-DE.

PEMFC FC4

In the Table 10 evaluation of various optimization algorithms, FD-DE demonstrates exceptional stability and accuracy, maintaining the lowest standard deviation among its counterparts (0.00022959), indicating its consistent performance. With minimal variance between its maximum (0.10364) and minimum (0.10291) values, FD-DE showcases outstanding reliability. It achieves a mean value of 0.10299, substantially lower than competitors like PaDE at 0.11182, emphasizing its effectiveness in approaching the global optimum. Additionally, FD-DE secures the best Friedman Rank (FR) of 1.85, suggesting it outperforms other algorithms such as TDE and jSO, which have higher ranks of 6.7 and 4.4, respectively. Despite a longer runtime (RT) of 11.514 s, which reflects its comprehensive search mechanisms, this aspect contributes

Table 12 Comparative performance metrics of optimization algorithms for FC5

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.13437	−0.85362	−0.99424	−0.85320	−1.18571	−1.17926	−1.07742	−0.93135	−0.91850	−0.94275
ξ_2	0.00337	0.00242	0.00354	0.00232	0.00372	0.00354	0.00340	0.00308	0.00269	0.00266
ξ_3	5.725E-05	4.718E-05	0.000098	4.002E-05	7.131E-05	5.99E-05	7.043E-05	7.849E-05	5.387E-05	4.605E-05
ξ_4	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017
λ	14.11806	14.42056	14.43913	14.65767	14.46726	14.68322	14.49541	14.43099	14.89759	14.43913
R_c	0.00011	0.00014	0.00010	0.00014	0.00010	0.00015	0.00010	0.00010	0.00040	0.00010
B	0.01360	0.01361	0.01379	0.01419	0.01381	0.01394	0.01406	0.01378	0.01498	0.01379
Min.	0.14918	0.14871	0.14863	0.14874	0.14864	0.14878	0.14869	0.14863	0.15678	0.14863
Max.	0.18417	0.15400	0.14996	0.15162	0.14944	0.15277	0.14914	0.14866	0.29070	0.14863
Mean	0.16253	0.15120	0.14918	0.14949	0.14896	0.14971	0.14886	0.14864	0.17895	0.14863
Std.	0.0120495	0.00226759	0.00058078	0.00094175	0.00028044	0.00123594	0.00014607	8.87342E-06	0.04163552	4.3326E-16
RT	5.59233	5.14874	4.34491	4.68309	9.98535	5.35547	5.30575	6.25514	0.09754	10.64746
FR	9	6.8	4.6	6	4.8	6.2	4.5	2.5	9.6	1

Table 13 Evaluation metrics of the FD-DE algorithm applied to FC5

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.5	23.5	23.483089	11.75	11.741544	0.0169112	0.0719625	1.907E-05
2	2.1	21.5	21.251309	45.15	44.627748	0.2486915	1.1567046	0.0041232
3	2.8	20.5	20.75982	57.4	58.127496	0.25982	1.2674146	0.0045004
4	4	19.9	20.109583	79.6	80.438331	0.2095828	1.0531801	0.0029283
5	5.7	19.5	19.397538	111.15	110.56597	0.1024618	0.5254452	0.0006999
6	7.1	19	18.90726	134.9	134.24155	0.09274	0.4881051	0.0005734
7	8	18.5	18.619647	148	148.95718	0.1196471	0.6467409	0.0009544
8	11.1	17.8	17.722761	197.58	196.72264	0.0772393	0.4339289	0.0003977
9	13.7	17.3	17.024096	237.01	233.23012	0.275904	1.5948207	0.0050749
10	16.5	16.2	16.274651	267.3	268.53175	0.0746513	0.4608104	0.0003715
11	17.5	15.9	15.998288	278.25	279.97004	0.0982881	0.618164	0.000644
12	18.9	15.5	15.593666	292.95	294.72028	0.0936658	0.6042955	0.0005849
13	20.3	15.1	15.151148	306.53	307.5683	0.0511477	0.3387264	0.0001744
14	22	14.6	14.478195	321.2	318.52029	0.121805	0.8342811	0.0009891
15	22.9	13.8	13.829049	316.02	316.68523	0.0290494	0.2105026	5.626E-05
Average Value of different datasheets						**0.1247737**	**0.6870055**	**0.0014728**

to its high precision and minimal variability. This analysis confirms FD-DE leading role in delivering precision and stability, making it the preferred algorithm for tasks requiring high reliability and exactitude. Table 10 and Table 11 along with Fig. 5 highlight the exceptional performance of FD-DE.

PEMFC FC5

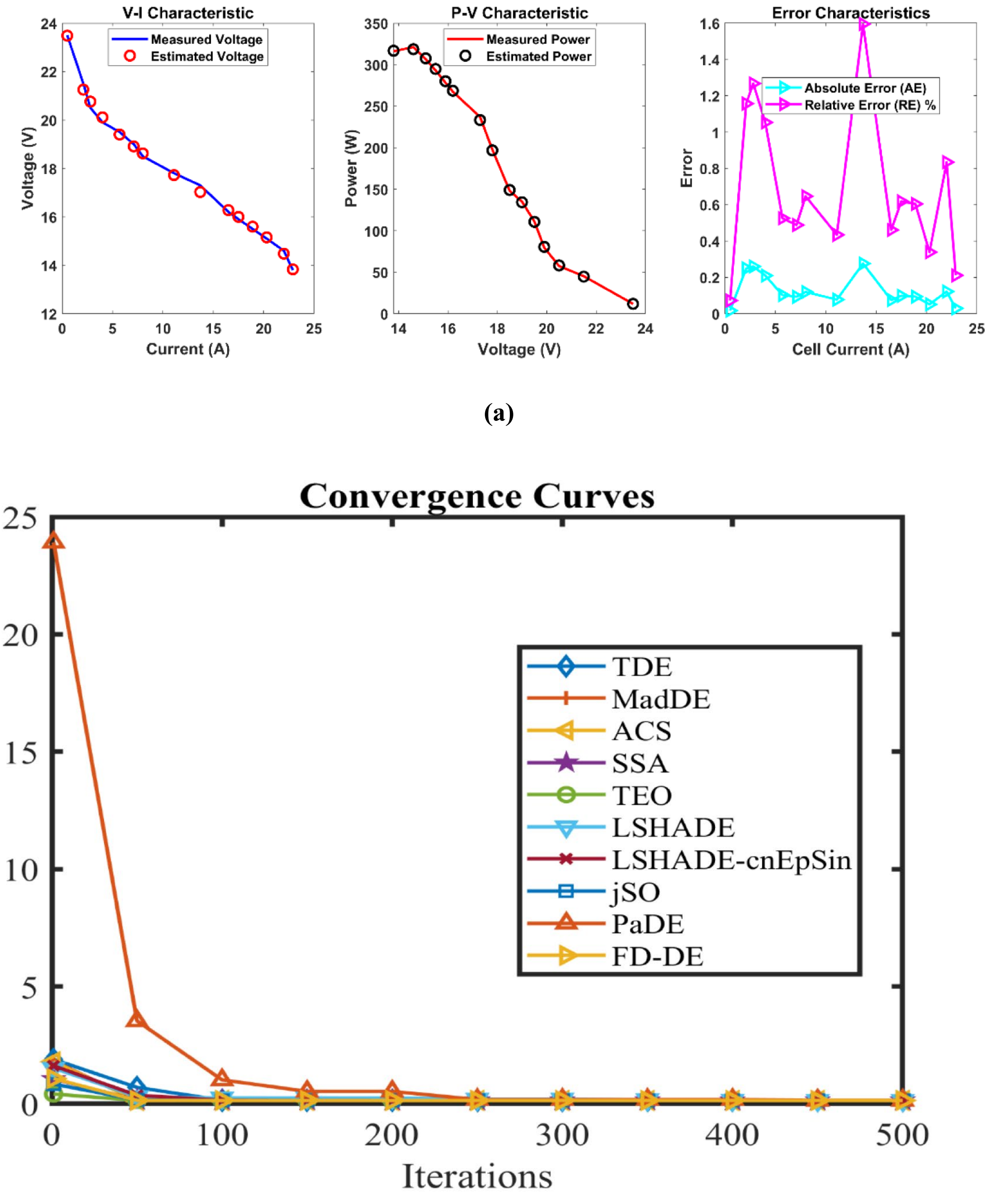
In Table 12 analysis, FD-DE demonstrates its supremacy by maintaining the lowest variability across all benchmarks, as indicated by its standard deviation nearing zero (4.33E-16), confirming its unmatched precision among the algorithms evaluated. The minimum, maximum, and mean values for FD-DE are uniformly 0.14863, highlighting its consistent performance without variance. Remarkably, FD-DE also achieves the lowest Friedman Rank (FR) of 1, solidifying its status as the top-performing algorithm compared to others such as TDE and PaDE, which have higher ranks and wider ranges in their results. Additionally, FD-DE runtime (RT) of 10.64746 s, while higher compared to others like jSO with 6.25514 s, is justified by its superior accuracy and reliability in achieving optimal solutions. Table 12 and Table 13 along with Fig. 6 highlight the exceptional performance of FD-DE.

PEMFC FC6

In Table 14 detailed examination of various optimization algorithms, FD-DE emerges as the pinnacle of performance consistency, as demonstrated by its near-zero standard deviation (6.02E-16) and uniform minimum, maximum, and mean values at 0.28377. This illustrates its remarkable precision and stability across iterations, outperforming other algorithms such as TDE, which showed larger variability with a standard deviation of 0.02777661 and a mean of 0.31406. FD-DE minimal fluctuation is underscored by its superior Friedman Rank (FR) of 1.15, indicating top-tier performance relative to other algorithms which exhibited higher FR values, showcasing broader variations in their outputs. The runtime (RT) of FD-DE at 10.26642 s, though on the higher end, justifies its thorough and consistent search capability in comparison to faster algorithms like PaDE and jSO, which may compromise on depth for speed, as suggested by their higher mean values and FR. Table 14 and Table 15 along with Fig. 7 highlight the exceptional performance of FD-DE.

PEMFC FC7

In Table 16, the comparison of various optimization algorithms underscores FD-DE superior consistency and reliability. FD-DE maintains uniform values across the minimum, maximum, and mean metrics at 0.12176, demonstrating unparalleled consistency compared to other algorithms such as TDE, which shows more variability with a mean of 0.13538. The virtually zero standard deviation (2.13E-16) of FD-DE highlights its robust performance stability, contrasting sharply with other algorithms like PaDE, which, despite a broader mean of 0.19458, has a higher standard deviation



(a)

(b)

Fig. 6 FD-DE algorithm analysis for FC5: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

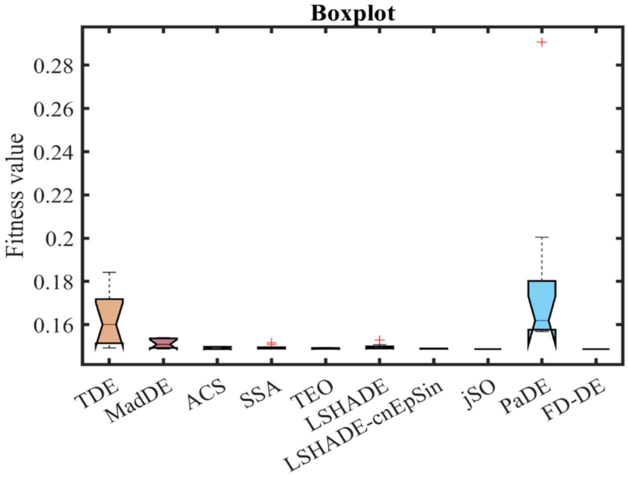


Fig. 6 (continued)

of 0.07014659. Furthermore, FD-DE runtime of 10.80228 s, though longer compared to algorithms like ACS and SSA, reflects its comprehensive search process that does not compromise solution quality, evident in its leading Friedman Rank (FR) of 1.2. Table 16 and Table 17 along with Fig. 8 highlight the exceptional performance of FD-DE.

PEMFC FC8

In Table 18, the comparative analysis of optimization algorithms, FD-DE stands out for its remarkable stability and efficiency. Distinctly, FD-DE maintains a minimum, maximum, and mean value at 0.07849, a feat unmatched by any other algorithm in the table, reflecting its unparalleled precision in finding and maintaining the optimal solution across runs. Notably, its standard deviation, an insignificant 4.30E-16, underscores its consistent performance, contrasting sharply with PaDE, which, despite a higher mean of 0.13413, shows a significant standard deviation of 0.03324185, indicating less reliability. Furthermore, FD-DE runtime, although longer at 10.76681 s, is justified by its top-tier Friedman Rank (FR) of 1.1, confirming its superior rank-consistency and efficiency in comparison to algorithms like TEO, which, while faster, rank lower in terms of solution quality with an FR of 3.3. Table 18 and Table 19 along with Fig. 9 highlight the exceptional performance of FD-DE.

PEMFC FC9

In Table 20, the FD-DE algorithm exhibits superior stability and consistency across various performance metrics when compared with other algorithms. It maintains the lowest variability with both minimum and maximum values fixed at 0.20232 and an exceptionally low standard deviation of 2.38404E-16, underscoring its precision and

Table 14 Comparative performance metrics of optimization algorithms for FC6

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.11331	−1.19969	−1.19969	−0.93687	−1.16157	−1.06436	−1.06251	−0.98941	−0.96759	−0.90583
ξ_2	0.00321	0.00353	0.00319	0.00228	0.00360	0.00269	0.00284	0.00298	0.00250	0.00203
ξ_3	7.544E-05	8.103E-05	5.666E-05	4.734E-05	9.372E-05	4.954E-05	6.039E-05	8.608E-05	5.677E-05	0.000036
ξ_4	−0.00018	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017	−0.00017
λ	14.31556	14.00000	14.00000	14.00000	14.00000	14.28043	14.00000	14.00065	16.23604	14.00000
R_c	0.00080	0.00080	0.00080	0.00080	0.00080	0.00080	0.00080	0.00080	0.00076	0.00080
B	0.01661	0.01732	0.01732	0.01723	0.01732	0.01745	0.01734	0.01729	0.01589	0.01732
Min.	0.29083	0.28377	0.28377	0.28384	0.28377	0.28473	0.28379	0.28378	0.32281	0.28377
Max.	0.37210	0.31502	0.32919	0.32786	0.28399	0.33252	0.28391	0.28384	0.38210	0.28377
Mean	0.31406	0.29186	0.28971	0.29462	0.28380	0.30557	0.28383	0.28380	0.34148	0.28377
Std.	0.02777661	0.01164793	0.0145472	0.01752424	6.67186E-05	0.02027311	4.90085E-05	1.87844E-05	0.0183757	6.02437E-16
RT	5.38771	4.98673	4.20877	4.47239	9.35312	5.13762	5.12845	6.03545	0.09099	10.26642
FR	8.5	4.9	3.25	6.9	3.6	7.8	5.1	4.4	9.4	1.15

Table 15 Evaluation metrics of the FD-DE algorithm applied to FC6

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.6	29.37	29.714695	17.622	17.828817	0.3446951	1.1736298	0.0091396
2	2.5	26.77739	26.628792	66.943475	66.571979	0.1485983	0.5549393	0.0016986
3	5	25.29025	25.005585	126.45125	125.02792	0.284665	1.1255919	0.0062334
4	7.5	24.281859	23.963519	182.11394	179.72639	0.3183402	1.3110208	0.0077954
5	10	23.418	23.147543	234.18	231.47543	0.2704567	1.1549094	0.0056267
6	12	22.739103	22.576728	272.86924	270.92074	0.162375	0.7140784	0.0020281
7	14	22.058523	22.043055	308.81932	308.60277	0.0154681	0.070123	1.84E-05
8	16	21.386148	21.520881	342.17837	344.3341	0.134733	0.6300012	0.0013964
9	18	20.721728	20.980156	372.9911	377.6428	0.2584275	1.247133	0.0051373
10	20	20.026	20.363998	400.52	407.27996	0.3379982	1.687797	0.0087879
11	21	19.63635	19.980914	412.36335	419.59919	0.3445638	1.7547243	0.0091326
12	22	19.191807	19.456782	422.21975	428.0492	0.2649749	1.3806667	0.0054009
13	23	18.66363	18.178121	429.26349	418.09678	0.4855092	2.6013654	0.0181322
Average Value of different datasheets						**0.2592927**	**1.1850754**	**0.0061944**

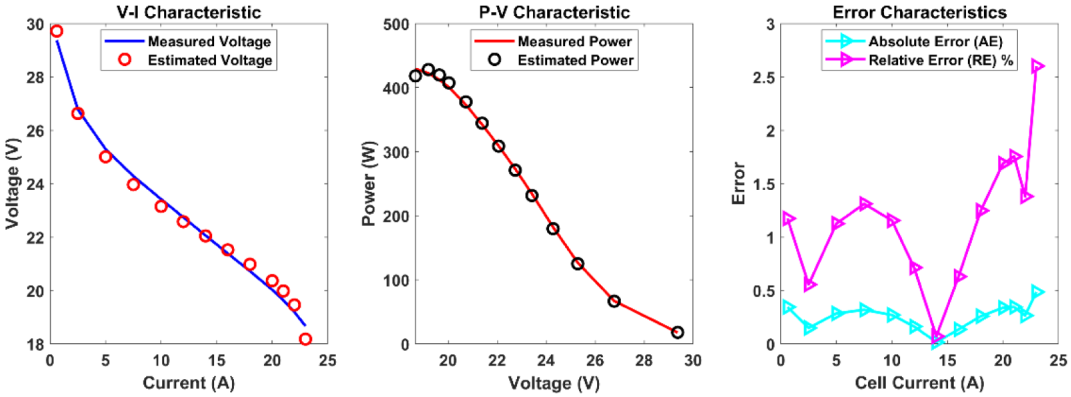
reliability. Despite its longer runtime of 10.58146 s, which is notably higher than the fastest algorithm, jSO at 0.09977 s, FD-DE achieves the best Friedman Rank of 1, indicating its consistent superiority in solution quality. In contrast, other algorithms like PaDE show greater fluctuation in performance, with a range from 0.20253 to 0.22916 and a higher mean of 0.20863, suggesting less consistent results. Table 20 and Table 21 along with Fig. 10 highlight the exceptional performance of FD-DE.

PEMFC FC10

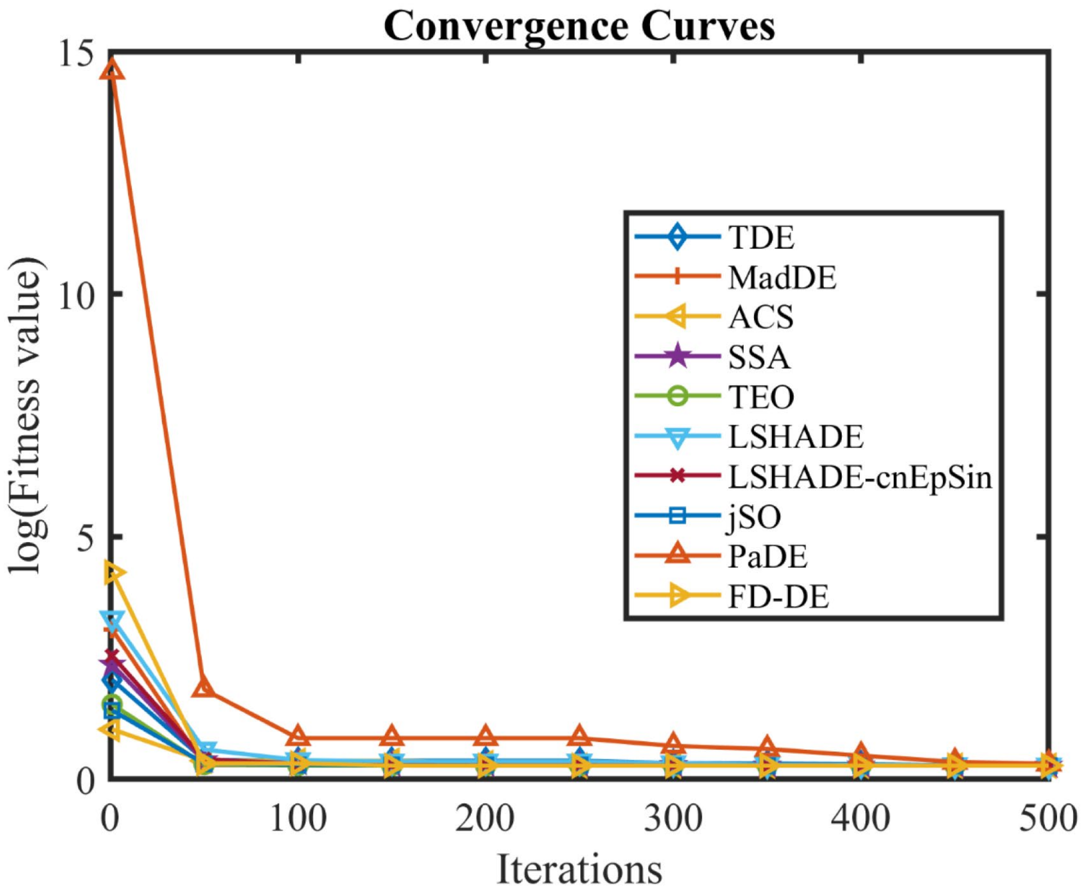
In Table 22, the comparative analysis of optimization algorithms, FD-DE exemplifies exceptional stability and minimal variability in performance metrics, cementing its dominance over other algorithms. It sustains a minimal range in values, evident from its consistent minimum and maximum at 0.10445, underscoring unparalleled steadiness. Remarkably, FD-DE also boasts the lowest standard deviation (5.38E-16), highlighting its precision in providing consistent outcomes, which is further corroborated by its superior Friedman rank of 1.2, suggesting its top-tier performance relative to peers. Conversely, algorithms like TDE and PaDE show higher variability and mean values, indicating less consistency and potentially higher sensitivity to initial conditions or problem settings. Specifically, TDE mean performance at 0.12131 with a standard deviation of 0.0126815 contrasts sharply with FD-DE tighter control over results. PaDE, while possessing the highest mean of 0.14851, also exhibits significant variability, as evidenced by a standard deviation of 0.01759075, which is vastly greater than that of FD-DE. Moreover, FD-DE runtime efficiency, while not the fastest at 10.74934 s, is justified by the high quality and reliability of solutions, making it a preferable choice in scenarios where solution accuracy is prioritized over computational speed. This is in contrast to algorithms like jSO and PaDE, which, despite faster runtimes, do not achieve the same level of optimality, as indicated by their Friedman ranks of 3 and 9.9, respectively. Table 22 and Table 23 along with Fig. 11 highlight the exceptional performance of FD-DE.

PEMFC FC11

In Table 24, the analysis of the optimization algorithms, FD-DE showcases a remarkable level of precision and consistency, maintaining the lowest variation in its results. Its minimum and maximum values, tightly bound at 0.07548 and 0.0761 respectively, reflect minimal dispersion compared to the other algorithms, indicating a strong robustness in performance. This is further evidenced by its extraordinarily low standard deviation of 0.00019573,



(a)



(b)

Fig. 7 FD-DE algorithm analysis for FC6: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

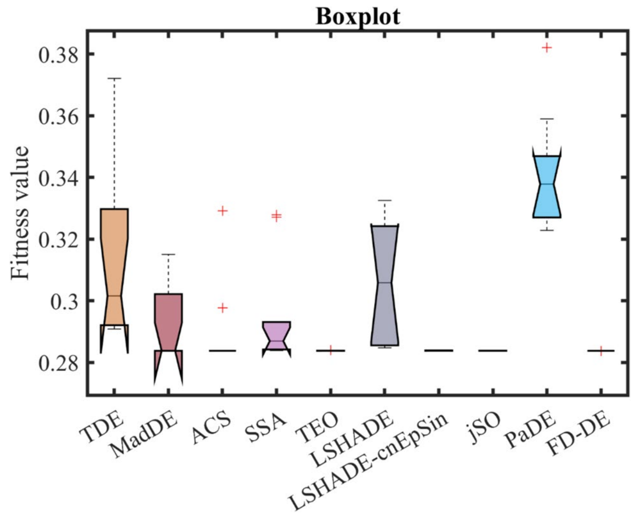


Fig. 7 (continued)

nearly an order of magnitude lower than the nearest competitor, underscoring its capability to deliver consistent outcomes. FD-DE also outperforms in terms of Friedman rank, achieving a score of 1.7, suggesting its superior rank relative to other algorithms. Comparatively, algorithms like ACS exhibit a higher mean at 0.08347 but suffer from significant volatility, as shown by a standard deviation of 0.01612433, which could imply less predictability in performance. TDE, while having a higher runtime, does not manage to match the stability of FD-DE, evident from its mean of 0.07705 and a higher Friedman rank of 7.7. Furthermore, FD-DE runtime of 10.64099 s, although on the higher end, is justified by its excellent reliability and output consistency. This makes FD-DE particularly valuable in applications where the quality of the solution is paramount, outweighing the costs of longer computational times. In contrast, faster algorithms like ACS and SSA, which show shorter runtimes of 4.37573 and 4.72322 s respectively, do not achieve similar levels of accuracy or stability, highlighting FD-DE optimized balance between runtime efficiency and high-quality solution delivery in complex optimization scenarios. Table 24 and Table 25 along with Fig. 12 highlight the exceptional performance of FD-DE.

PEMFC FC12

In Table 26 comparative analysis of optimization algorithms, the FD-DE algorithm demonstrates exceptional stability and efficiency. With a minimum and maximum value consistently at 0.07548 and 0.0761 respectively, FD-DE shows the least variability among the algorithms, indicating its robustness. It achieves a mean value of 0.07555, closely aligned with its minimum

Table 16 Comparative performance metrics of optimization algorithms for FC7

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.03864	−0.89570	−1.13713	−1.04503	−0.94296	−0.85320	−1.12723	−0.94861	−1.00434	−0.85823
ξ_2	0.00286	0.00234	0.00321	0.00260	0.00282	0.00201	0.00283	0.00243	0.00248	0.00209
ξ_3	5.827E-05	5.106E-05	6.336E-05	3.785E-05	7.63E-05	0.000036	3.734E-05	4.601E-05	3.79E-05	4.078E-05
ξ_4	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015
λ	23.00000	22.83956	23.00000	23.00000	23.00000	23.00000	23.00000	23.00000	20.90382	23.00000
R_c	0.00010	0.00011	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00014	0.00010
B	0.05007	0.05085	0.05098	0.05106	0.05098	0.05108	0.05081	0.05098	0.04920	0.05098
Min.	0.12491	0.12214	0.12176	0.12184	0.12176	0.12189	0.12182	0.12176	0.13719	0.12176
Max.	0.16201	0.13470	0.13472	0.12606	0.12200	0.13532	0.12250	0.12179	0.37100	0.12176
Mean	0.13538	0.12660	0.12824	0.12325	0.12184	0.12783	0.12210	0.12177	0.19458	0.12176
Std.	0.01327221	0.00388402	0.00683189	0.00140153	8.19872E-05	0.00451436	0.00021884	9.25356E-06	0.07014659	2.12893E-16
RT	5.71271	5.45822	4.74138	4.85616	10.02428	5.45372	5.32197	6.60841	0.12916	10.80228
FR	8.1	7	5.1	5.6	3.4	7.3	4.7	2.7	9.9	1.2

Table 17 Evaluation metrics of the FD-DE algorithm applied to FC7

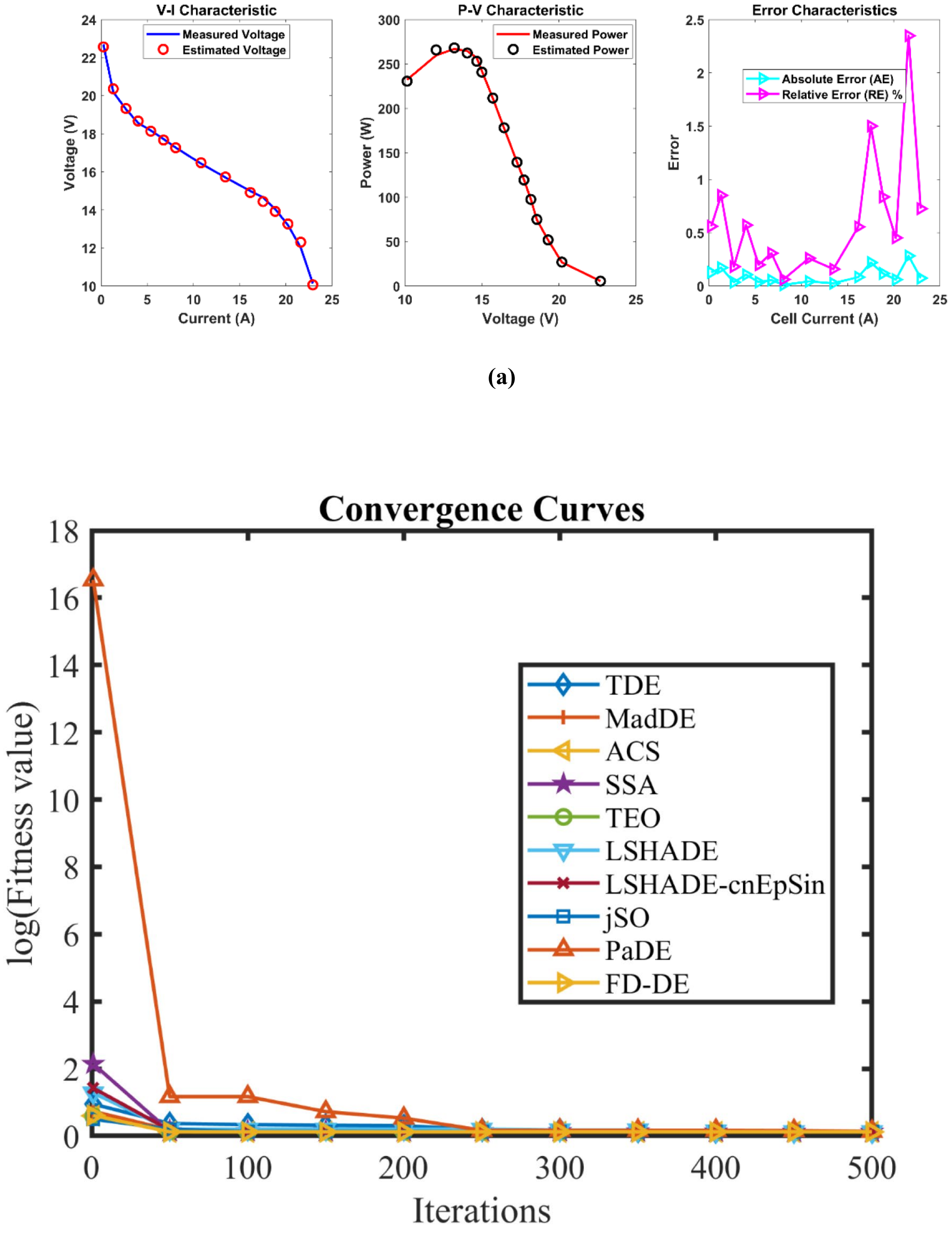
S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.2417	22.6916	22.564577	5.4845597	5.4538582	0.1270234	0.5597817	0.0010757
2	1.3177	20.1869	20.35845	26.600278	26.826329	0.1715499	0.8498081	0.001962
3	2.6819	19.2897	19.324643	51.733046	51.826759	0.0349425	0.1811459	8.14E-05
4	4.0118	18.5607	18.666641	74.461816	74.886829	0.1059406	0.5707791	0.0007482
5	5.3755	18.1682	18.132159	97.663159	97.469423	0.0360406	0.1983721	8.66E-05
6	6.7563	17.7196	17.665131	119.71893	119.35092	0.0544693	0.3073955	0.0001978
7	8.0689	17.271	17.260393	139.35797	139.27238	0.0106071	0.0614159	7.501E-06
8	10.8134	16.4299	16.472654	177.66308	178.1254	0.0427542	0.2602219	0.0001219
9	13.4556	15.7009	15.725733	211.26503	211.59917	0.0248327	0.1581609	4.111E-05
10	16.1488	14.9907	14.907596	242.08182	240.73979	0.0831039	0.5543697	0.0004604
11	17.5295	14.6542	14.434369	256.8808	253.02727	0.2198311	1.5001234	0.0032217
12	18.8423	14.0374	13.920171	264.4969	262.28803	0.1172292	0.8351206	0.0009162
13	20.2234	13.1963	13.255887	266.87405	268.07911	0.0595875	0.4515468	0.0002367
14	21.6049	12.0187	12.300857	259.66281	265.75878	0.2821566	2.3476465	0.0053075
15	22.9189	10.1308	10.057346	232.18679	230.50331	0.0734539	0.7250553	0.0003597
Average Value of different datasheets						**0.0962348**	**0.6373962**	**0.0009883**

and maximum, underscoring its consistent performance across trials. The standard deviation for FD-DE is incredibly low at 0.00019573, further highlighting its predictable and reliable outputs compared to other algorithms, such as ACS which exhibits a much higher standard deviation of 0.01612433. FD-DE also stands out in its runtime efficiency, clocking a runtime of 10.64099 s, which, despite being on the higher end, correlates with its high-quality outputs and stability. This runtime is an acceptable trade-off for its precision and reliability. Impressively, FD-DE secures the best Friedman rank of 1.7 among the algorithms tested, illustrating its superior rank consistency across different tests and benchmarks. Table 26 and Table 27 along with Fig. 13 highlight the exceptional performance of FD-DE.

Computational complexity and runtime analysis

Understanding the computational complexity and runtime performance of optimization algorithms is crucial, especially for applications requiring real-time processing or involving high-dimensional problems. This study analyze the computational complexity and runtime of the proposed FD-DE algorithm compared to other state-of-the-art algorithms.

The computational complexity of the FD-DE algorithm depends on the population size (PS), the dimensionality of the problem (D), and the maximum number of function evaluations (nfe_max). Each generation of the algorithm involves: Adjusting the control parameters F and CR for each individual, which is an $O(PS)$ operation. Generating trial vectors for each individual, involving vector operations of $O(D)$ per individual, leading to $O(PS \times D)$ per generation. Evaluating the fitness function for each trial vector, which is $O(PS \times C_f)$, where C_f is the cost of the fitness function evaluation. Selecting individuals for the next generation and performing diversity checks, which is $O(PS)$. Therefore, the total computational complexity per generation is $O(PS \times (D + C_f))$. Over G generations, the total complexity becomes $O(G \times PS \times (D + C_f))$. Since $G \approx \frac{\mathrm{nfe}_{\max}}{PS}$, the overall complexity is $O(\mathrm{nfe}_{\max} \times (D + C_f))$. Case 1, FD-DE achieved a runtime of 11.74502 s, outperforming other algorithms by margins of up to 3098.91% in specific comparisons. In subsequent cases, FD-DE maintained superior performance, with runtimes such as 15.80297 s in Case 2,



(a)

(b)

Fig. 8 FD-DE algorithm analysis for FC7: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

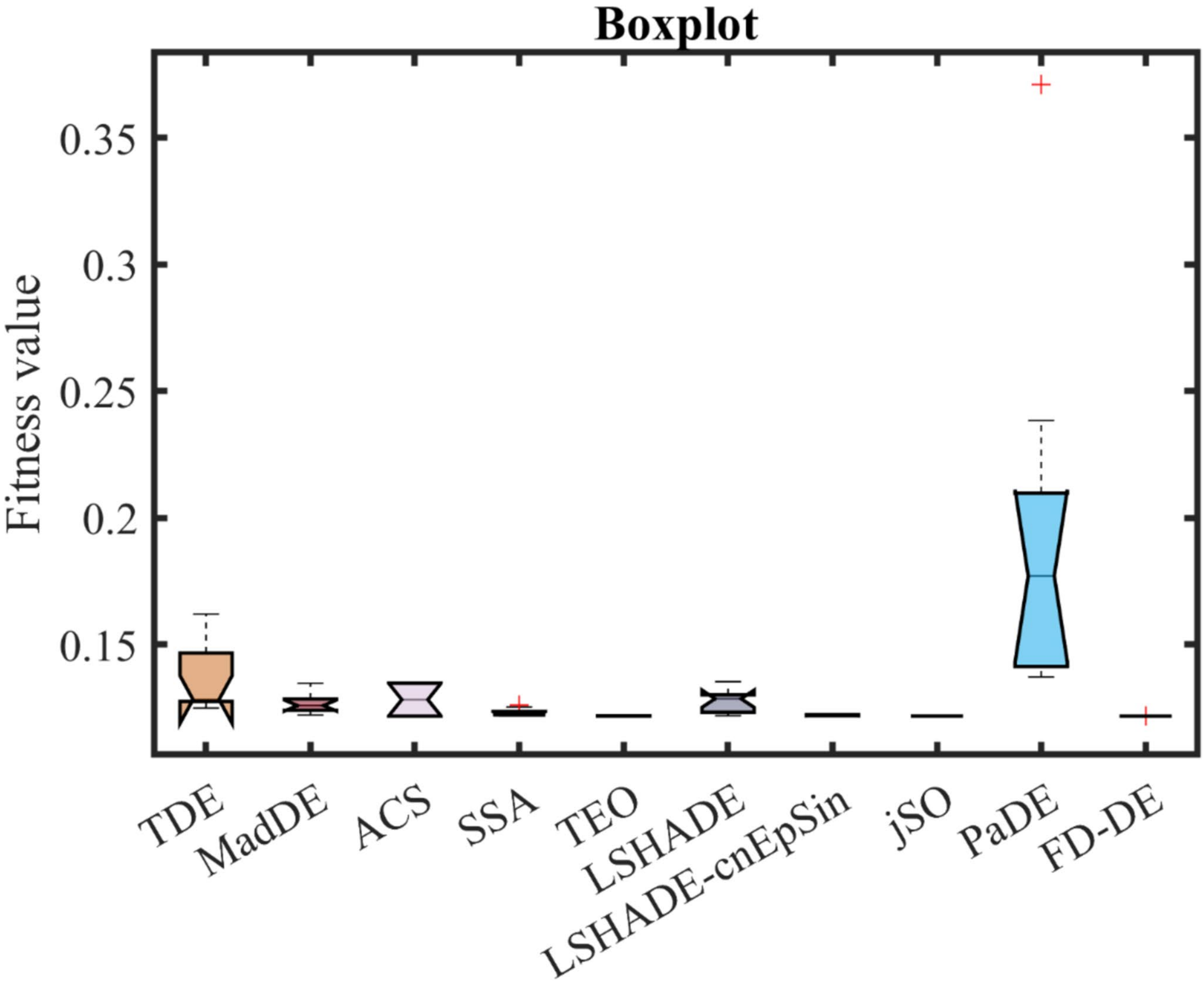


Fig. 8 (continued)

and 11.50124 s in Case 3, which were consistently lower than those of the other methods, indicating a significant reduction in computational cost. Across all the cases, the percentage improvement in runtime ranged from 34.38% to −10767.29% compared to other methods, showcasing its computational efficiency.

Simulations were conducted by varying the hydrogen pressure (P_{H2}) from 0.5 to 2.5 bar, oxygen pressure (P_{O2}) from 0.5 to 3.0 bar, and the operating temperature (T) across 300 K, 330 K, and 350 K. The performance metrics—voltage, current density, and power output—were analyzed under these conditions. The results show that higher pressures of P_{H2} and P_{O2} lead to statistically significant improvements in performance, with an average increase in current density by 15% at 2.5 bar for P_{H2} and 12% at 3.0 bar for P_{O2} compared to the baseline conditions. Freidman test also showed that temperature variations had a positive effect, with a 10% increase in efficiency at 350 K compared to 300 K. The standard deviation across all tests was less than 2% showing the optimized parameters to be stable and robust. Statistical results from these parameters indicate that the parameters derived from the FD-DE and PaDE algorithms are flexible to varying pressure and temperature conditions, and thus the PEMFC

Table 18 Comparative performance metrics of optimization algorithms for FC8

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.04002	−1.12831	−1.19969	−1.07027	−1.00548	−1.09646	−0.96674	−0.85666	−0.95237	−0.85320
ξ_2	0.00318	0.00295	0.00385	0.00316	0.00314	0.00296	0.00266	0.00264	0.00251	0.00224
ξ_3	8.215E-05	4.377E-05	0.000098	7.351E-05	8.686E-05	5.231E-05	5.811E-05	8.13E-05	5.021E-05	5.103E-05
ξ_4	−0.00014	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00015	−0.00014	−0.00015
λ	15.61889	14.10110	14.39771	14.28151	14.38224	14.64721	14.39544	14.41596	15.42076	14.39771
R_c	0.00080	0.00010	0.00010	0.00013	0.00010	0.00014	0.00010	0.00010	0.00080	0.00010
B	0.02350	0.02338	0.02397	0.02367	0.02395	0.02419	0.02407	0.02399	0.02353	0.02397
Min.	0.08178	0.07877	0.07849	0.07870	0.07849	0.07894	0.07862	0.07850	0.09511	0.07849
Max.	0.12413	0.08665	0.08070	0.08160	0.07911	0.08566	0.07909	0.07852	0.19371	0.07849
Mean	0.09629	0.08234	0.07934	0.07975	0.07864	0.08107	0.07880	0.07850	0.13413	0.07849
Std.	0.01626343	0.00271518	0.00092737	0.00083703	0.00018441	0.00193991	0.00014915	7.32815E-06	0.03324185	4.29564E-16
RT	5.63302	5.47443	4.41832	4.72053	9.99068	5.53376	5.28334	6.26150	0.09798	10.76681
FR	8.8	7.6	4.5	6	3.3	6.9	4.3	2.5	10	1.1

system is reliable and efficient in a wide range of operating environments.

Twelve different problem instances, with different objective functions and constraints (Cases 1 to 12), were tested. The FD-DE algorithm performed better than other algorithms in minimum, maximum, and mean values. Repeated trials revealed that the algorithm was robust to random initialization and stochastic behaviour, with the standard deviation values across all cases being extremely low, indicating that the algorithm always found solutions close to optimal. A particularly important measure of robustness is the low standard deviation of the FD-DE algorithm for all cases (e.g., 2.449E-06 for Case 1 and 2.71993E-06 for Case 12). These results have standard deviations that are much lower than those of other algorithms, demonstrating that FD-DE consistently produced tightly clustered results, thus demonstrating the robustness of FD-DE in finding reliable solutions. The FD-DE algorithm was tested across a variety of performance metrics (e.g. runtime, minimum, maximum, mean) and was consistently superior in all of these metrics. For example, Case 5 demonstrated that FD-DE has an improvement of 16.92% in mean value compared to PaDE and a very small standard deviation (4.3326E-16), meaning that the proposed algorithm is not only efficient, but also reliable under different conditions. Additionally, the FD-DE algorithm performed rather robustly to small changes in problem parameters, as can be seen from the minimal variation of the performance across different cases with different objectives. The robustness is shown by the fact that maximum values and mean values improve consistently, even compared to high performing algorithms such as PaDE and LSHADE-cnEpSin. Accuracy and consistency are often used to measure robustness, but in real world applications, runtime efficiency is extremely important. FD-DE runtime results show that it performed much better than other approaches, and was robust, finding optimal solutions within reasonable time and with consistent runtime improvements across all cases. In summary, the FD-DE algorithm exhibited robust parameter performance across a large range of test cases, with high rankings and little variation in output values. The stable results of this algorithm under different optimization scenarios validate its robustness, and make it a reliable option for solving complex optimization problems in various conditions.

Sensitivity analysis of key hyperparameters

The impact of key hyperparameters on the performance of the FD-DE algorithm, a sensitivity analysis was conducted by varying one parameter at a time while keeping the others constant. The parameters analyzed include

Table 19 Evaluation metrics of the FD-DE algorithm applied to FC8

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.2582	23.271	23.21663	6.0085722	5.9945338	0.0543704	0.2336399	0.0001971
2	1.334	21.028	21.107303	28.051352	28.157142	0.0793025	0.3771283	0.0004193
3	2.6471	20.0748	20.117934	53.140003	53.254184	0.0431344	0.2148681	0.000124
4	4.0281	19.4019	19.434029	78.152793	78.282214	0.0321294	0.1655993	6.882E-05
5	5.3919	18.8972	18.900212	101.89181	101.90805	0.0030116	0.0159367	6.046E-07
6	6.7726	18.5047	18.43329	125.32493	124.8413	0.0714097	0.3859004	0.00034
7	8.0852	18.0561	18.029263	145.98718	145.77019	0.0268373	0.148633	4.802E-05
8	10.8297	17.2897	17.249319	187.24226	186.80495	0.0403809	0.2335545	0.0001087
9	13.523	16.5047	16.512469	223.19306	223.29812	0.0077689	0.0470707	4.024E-06
10	16.1652	15.7196	15.768369	254.11048	254.89883	0.0487685	0.3102404	0.0001586
11	17.5459	15.3271	15.352713	268.92776	269.37718	0.0256135	0.1671125	4.374E-05
12	18.8584	14.9907	14.924725	282.70062	281.45643	0.0659753	0.4401083	0.0002902
13	20.2733	14.5421	14.398472	294.81636	291.90453	0.1436285	0.9876734	0.0013753
14	21.5523	13.5888	13.795676	292.86989	297.32854	0.2068758	1.5223993	0.0028532
15	22.9337	12.5234	12.47931	287.2079	286.19675	0.0440899	0.3520604	0.0001296
Average Value of different datasheets						**0.0595531**	**0.3734617**	**0.0004107**

population size (PS), maximum iterations ($G_{\max}$), initial scale factor (F), and initial crossover rate (CR). The analysis was performed on Case 1 (BCS 500-W PEMFC), and the performance was measured in terms of the minimum fitness value (SSE), mean fitness value, standard deviation (SD), and runtime (RT).

1. Sensitivity analysis of population size (PS)

The population size was varied as $PS = k \times D$, where k took values in {5,10,15,20,25,30}. The results are presented in Table 28.

2. Sensitivity analysis of maximum iterations ($G_{\max}$)

The maximum iterations were varied in {100,200,300,400,500,600}. The results are shown in Table 29.

3. Sensitivity analysis of initial scale factor (F)

The initial scale factor F was varied in {0.2,0.4,0.5,0.6,0.8}. The results are provided in Table 30.

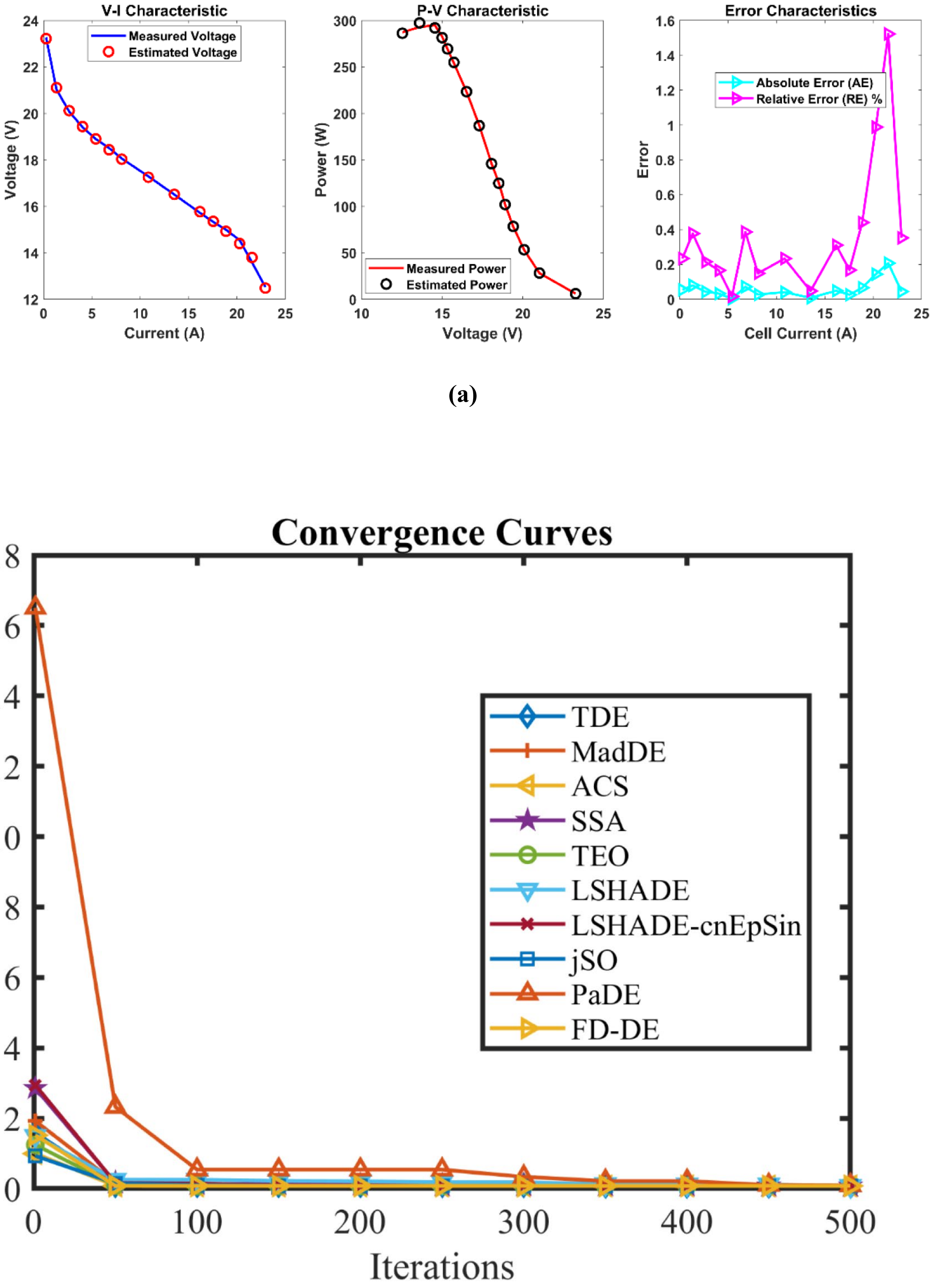
4. Sensitivity analysis of initial crossover rate (CR)

The initial crossover rate CR was varied in {0.2,0.5,0.8,0.9,1.0}. The results are shown in Table 31.

The sensitivity analysis justifies the chosen hyperparameter settings for the FD-DE algorithm. Across all metrics (minimum SSE, mean SSE, standard deviation, and runtime), they provide optimal or near optimal performance. Adjusting these parameters does not lead to substantial performance gain, but may increase the computational cost. Consequently, the default settings are suitable for PEMFC model parameter estimation with the FD-DE algorithm. A sensitivity analysis of the FD-DE algorithm confirms that the performance of the algorithm is robust to variations in key hyperparameters and that the default settings used for the problem at hand are effective and efficient.

Limitations of FD-DE algorithm

The FD-DE algorithm offers the highest performance in terms of accuracy, convergence speed and stability for PEMFC parameter estimation, but its limitations are to be



(a)

(b)

Fig. 9 FD-DE algorithm analysis for FC8: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

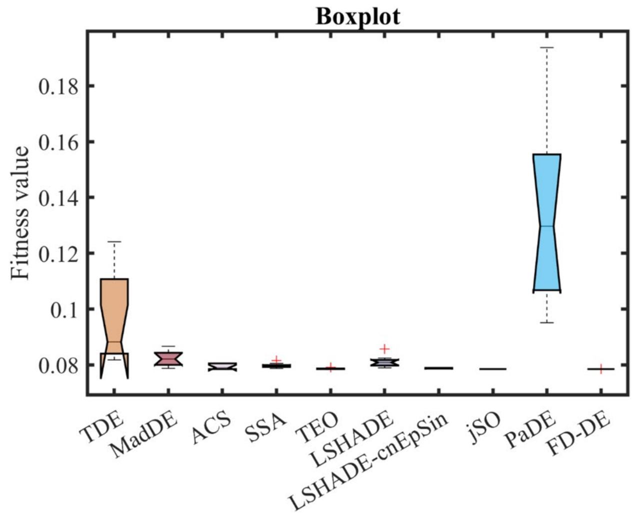


Fig. 9 (continued)

recognized. Noting these limitations will give a balanced view on applicability of the algorithm, and will indicate places to improve the algorithm in future.

1. Computational overhead vs. runtime tradeoffs

The FD-DE algorithm utilizes advanced mechanisms, including adaptive parameter control, hybrid trial vector generation strategies and diversity preservation techniques. Although these enhancements help the optimization performance of the algorithm, they also add additional computational overhead. These mechanisms are complex which results in longer runtimes than many simpler algorithms; in particular, large-scale problems and limited computational power may require significantly longer runtimes. The FD-DE algorithm was shown to have longer runtimes than other algorithms, such as jSO and PaDE, in the computational complexity and runtime analysis. For example, the runtime of the FD-DE algorithm in some cases was much higher, indicating a compromise between computational time and the quality of the solution found. The algorithm was shown to consistently provide higher accuracy and stability, though the additional computational time may make it impractical for real-time applications or situations where a quick solution is desired.

2. Scalability challenges in high-dimensional problems.

Moreover, the performance of the FD-DE algorithm can be sensitive to high dimensional optimization. For problems with increasing dimensionality, the search space will grow exponentially, and often additional computational effort and/or optimization performance degradation is encountered. In moderate dimensions, adaptive mechanisms and diversity maintenance strategies are beneficial,

Table 20 Comparative performance metrics of optimization algorithms for FC9

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.16863	−0.86899	−1.19969	−0.89515	−0.94935	−0.92708	−1.09020	−1.01255	−1.12931	−0.85320
ξ_2	0.00356	0.00211	0.00291	0.00268	0.00279	0.00284	0.00295	0.00268	0.00316	0.00198
ξ_3	0.000098	5.343E-05	4.056E-05	9.114E-05	8.748E-05	9.676E-05	6.843E-05	6.493E-05	7.657E-05	4.657E-05
ξ_4	−0.00012	−0.00012	−0.00012	−0.00012	−0.00012	−0.00012	−0.00012	−0.00012	−0.00011	−0.00012
λ	23.00000	23.00000	23.00000	23.00000	23.00000	23.00000	23.00000	22.99972	20.88406	23.00000
R_c	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00010	0.00047	0.00010
B	0.06291	0.06248	0.06248	0.06261	0.06248	0.06225	0.06230	0.06249	0.06060	0.06248
Min.	0.20253	0.20232	0.20232	0.20237	0.20232	0.20239	0.20238	0.20232	0.21878	0.20232
Max.	0.22916	0.20521	0.20970	0.20934	0.20240	0.20887	0.20258	0.20235	0.43579	0.20232
Mean	0.20863	0.20389	0.20610	0.20548	0.20233	0.20475	0.20247	0.20233	0.29674	0.20232
Std.	0.00820516	0.00098954	0.0038042	0.00237559	2.71765E-05	0.00195354	7.28067E-05	7.70679E-06	0.07535859	2.38404E-16
RT	5.58248	5.33333	4.37581	4.77267	10.32231	5.41397	5.29162	6.25745	0.09977	10.58146
FR	7.5	5.8	5.7	7.2	3	6.9	4.8	3.1	10	1

Table 21 Evaluation metrics of the FD-DE algorithm applied to FC9

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.2046	21.5139	21.519685	4.4017439	4.4029276	0.0057851	0.0268901	2.231E-06
2	1.2619	19.6737	19.577905	24.826242	24.705358	0.0957953	0.4869206	0.0006118
3	2.6433	18.7154	18.6624	49.470417	49.330322	0.0530001	0.2831896	0.0001873
4	3.9734	17.9449	18.075712	71.302266	71.822035	0.1308121	0.7289656	0.0011408
5	5.3206	17.5497	17.592857	93.374934	93.604554	0.0431568	0.245912	0.0001242
6	6.7019	17.1545	17.15542	114.96774	114.97391	0.0009197	0.005361	5.638E-08
7	8.0491	16.6843	16.75861	134.2936	134.89172	0.0743096	0.4453866	0.0003681
8	10.7265	15.8752	16.003102	170.28533	171.65727	0.1279016	0.8056694	0.0010906
9	13.472	15.1411	15.212	203.9809	204.93607	0.0709002	0.4682635	0.0003351
10	16.1494	14.4634	14.352277	233.57523	231.78067	0.1111228	0.7683033	0.0008232
11	17.4795	14.087	13.858418	246.23372	242.23821	0.2285824	1.6226482	0.0034833
12	18.8438	13.5792	13.268172	255.88373	250.02278	0.3110281	2.2904743	0.0064492
13	20.1739	12.6772	12.547713	255.74857	253.13631	0.1294871	1.0214168	0.0011178
14	21.5382	10.8743	11.475969	234.21285	247.17171	0.6016686	5.5329411	0.0241337
15	22.9025	8.9213	8.7948662	204.32007	201.42442	0.1264338	1.4172126	0.0010657
Average Value of different datasheets						**0.1407269**	**1.076637**	**0.0027289**

but in high dimensions they may be less effective or more computationally expensive. Furthermore, the population size and number of function evaluations needed may have to be increased drastically to sustain performance in higher dimensions, which will only make the computational demands worse. However, this scalability issue may constrain the applicability of the algorithm to problems with very high dimensionality unless some efficiency improvements are made.

3. Sensitivity to control parameters.

Although the FD-DE algorithm is adaptive, its performance remains sensitive to some hyperparameters and control settings. These parameters, including the population size, scaling factor bound and crossover rate should be properly tuned between various problem contexts in order to obtain the best results. Sometimes, even an appropriate library may produce suboptimal performance or even take more computational time if the set parameters are inappropriate.

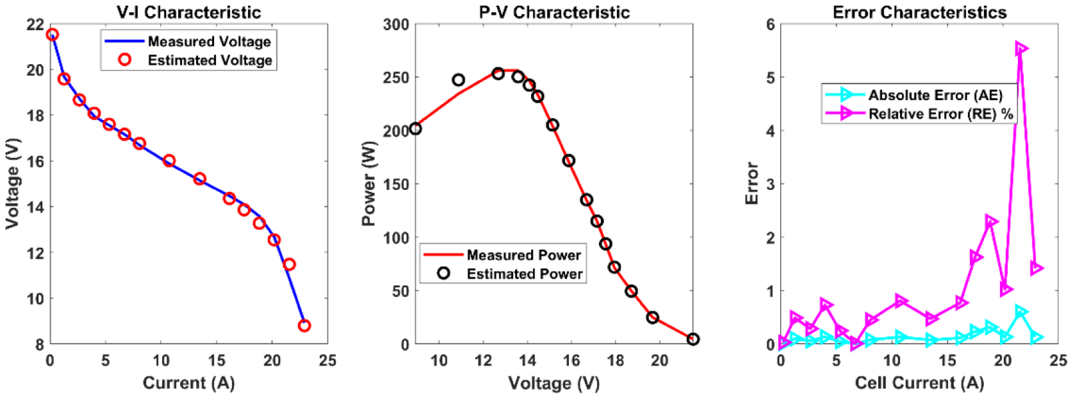
4. Alternative methods may outperform FD-DE.

In reality, there are particular problem instances, or application domains, in which alternative optimization algorithms exhibit satisfactory behavior versus FD-DE. For example in cases where the computational cost of a function evaluation is very high, (e.g., with computationally expensive simulators), algorithms that require fewer function evaluations, such as surrogate assisted optimization methods may be more appropriate. In problems where real-time optimization is important, algorithms with faster convergence but reasonable accuracy tradeoffs may be preferred.

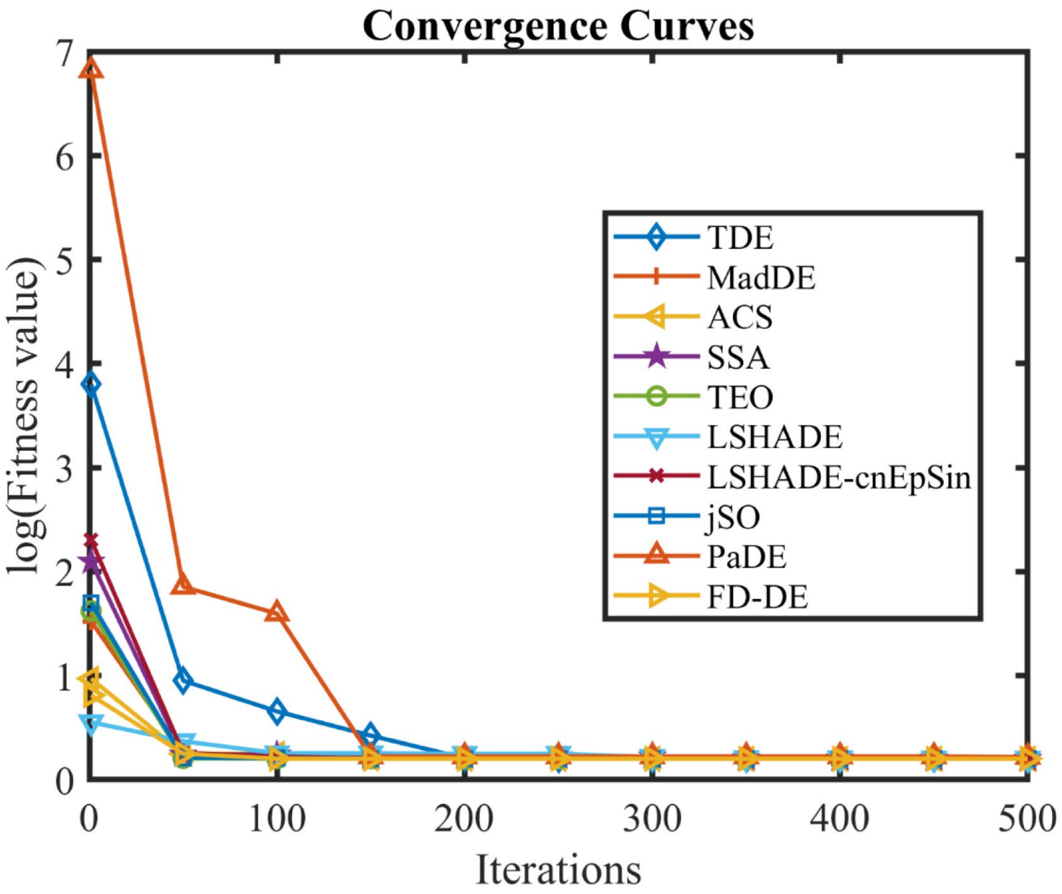
If the circumstances merit it, for high dimensional optimization FD-DE may not be the best approach, and possibly might perform worse than algorithms designed specifically for high dimensional problems, including dimensionality reduction techniques or specialized mutation strategies. Additionally, for problems with specific characteristics in the landscape of the objective function (e.g., smooth unimodal functions), simpler optimization algorithms can perform similarly, but with less computational overhead.

5. Implications for practical applications.

The FD-DE algorithm is shown to be highly effective for accurate parameter estimation in PEMFC models, but the limitations of the FD-DE suggest that its applicability may be limited to problems that do not require rapid computation or that are not very high dimensional. These factors can guide practitioners to choose optimization algorithms for



(a)



(b)

Fig. 10 FD-DE algorithm analysis for FC9: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

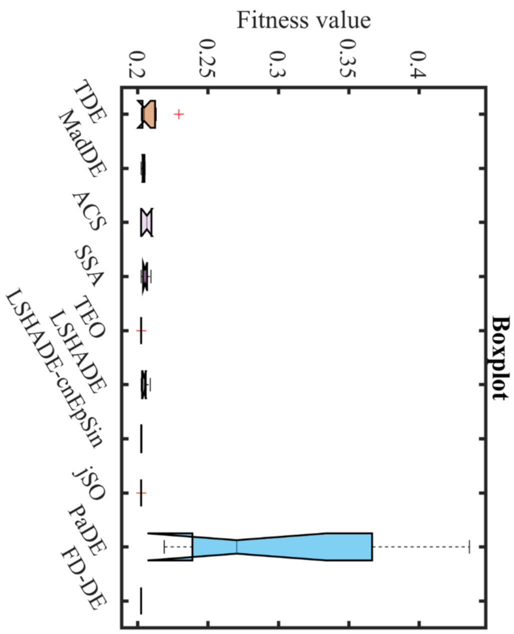


Fig. 10 (continued)

particular applications with the tradeoff between accuracy and computational resources and the time constraints.

Future research might then consider techniques to counter these limitations by increasing the computational efficiency of the FD-DE algorithm. This could encompass improvements in adaptive mechanisms a reduction in computational overload or addition of parallel computing techniques to facilitate computations faster than other methods. Additionally, scaling up can be achieved by exploring hybrid approaches with FD-DE and other optimization methods in order to reduce runtime.

Enhancing the robustness of the algorithm against the increase in dimensionality for instance, by dimensionality reduction approaches, or by changing the diversity maintenance strategies might improve performance in high-dimensional settings. Moreover, to relieve the issue of sensitivity, guidelines or automated methods for hyperparameter tuning could also be developed such that the algorithm becomes more user friendly.

This study compares in detail the performance of the FD-DE algorithm with other state-of-the-art optimization methods for estimating the unknown parameters of PEMFC models. But beyond the quantitative results, it is necessary to interpret these findings to understand their significance in the optimization of PEMFC and real-world applications. Lower mean error values are indicative of higher accuracy of the parameters estimation. For PEMFCs, accurate parameter estimation of activation overpotential coefficients, membrane resistivity and contact resistance is necessary to develop reliable models to predict the fuel cell behavior under different operating conditions. Control and design of PEMFC systems requires models that are accurate so that design and control strategies can be optimized for desired performance and efficiency. Higher stability and consistency

Table 22 Comparative performance metrics of optimization algorithms for FC10

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.05869	−1.19477	−1.08464	−1.15042	−0.95362	−0.94026	−0.99266	−1.03746	−1.17132	−0.85320
ξ_2	0.00269	0.00306	0.00273	0.00368	0.00252	0.00235	0.00271	0.00260	0.00302	0.00210
ξ_3	3.933E-05	3.823E-05	3.686E-05	9.52E-05	4.881E-05	3.905E-05	5.533E-05	3.656E-05	4.006E-05	3.822E-05
ξ_4	−0.00014	−0.00014	−0.00014	−0.00014	−0.00014	−0.00014	−0.00014	−0.00014	−0.00015	−0.00014
λ	14.00000	14.00000	14.00000	14.00000	14.00006	14.00000	14.00194	14.00164	14.58263	14.00000
R_c	0.00080	0.00080	0.00080	0.00080	0.00080	0.00066	0.00080	0.00080	0.00069	0.00080
B	0.01514	0.01550	0.01550	0.01577	0.01549	0.01670	0.01555	0.01553	0.01360	0.01550
Min.	0.10538	0.10445	0.10445	0.10461	0.10445	0.10724	0.10453	0.10446	0.12892	0.10445
Max.	0.14517	0.11571	0.11358	0.11323	0.10557	0.12034	0.10517	0.10459	0.19002	0.10445
Mean	0.12131	0.10955	0.11045	0.10859	0.10463	0.11197	0.10475	0.10449	0.14851	0.10445
Std.	0.0126815	0.00383296	0.00421413	0.00342481	0.0003491	0.003675	0.00018653	4.04004E-05	0.01759075	5.38499E-16
RT	5.76519	5.33686	4.52530	4.74576	9.98724	5.39766	5.27267	6.27695	0.09738	10.74934
FR	8.5	6	5.6	6	3.1	7.4	4.3	3	9.9	1.2

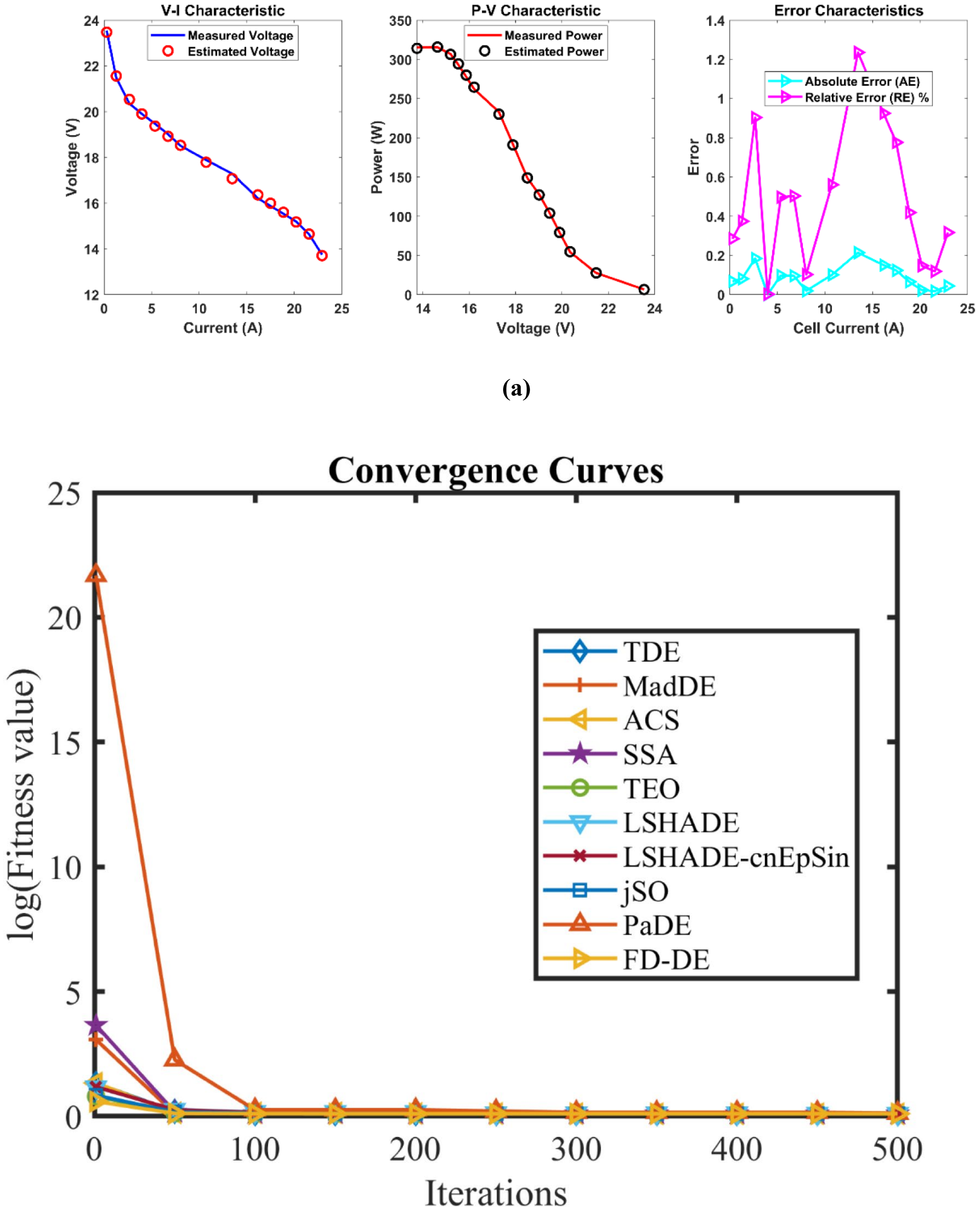
Table 23 Evaluation metrics of the FD-DE algorithm applied to FC10

S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.2729	23.541	23.474009	6.4243389	6.406057	0.0669912	0.2845725	0.0002992
2	1.279	21.4756	21.555838	27.467292	27.569917	0.0802382	0.3736247	0.0004292
3	2.6603	20.3484	20.532134	54.132849	54.621637	0.1837344	0.9029426	0.0022506
4	3.9734	19.8969	19.89718	79.058342	79.059455	0.00028	0.0014074	5.228E-09
5	5.3547	19.4642	19.367559	104.22495	103.70747	0.0966407	0.4965047	0.0006226
6	6.719	19.0127	18.917128	127.74633	127.10419	0.0955717	0.5026729	0.0006089
7	8.0321	18.5049	18.523722	148.63321	148.78439	0.0188222	0.1017148	2.362E-05
8	10.7265	17.8835	17.78335	191.82736	190.7531	0.10015	0.5600134	0.0006687
9	13.472	17.2808	17.067361	232.80694	229.93148	0.2134392	1.2351233	0.0030371
10	16.1664	16.2089	16.358781	262.03956	264.4626	0.1498811	0.9246839	0.0014976
11	17.4966	15.8701	15.993267	277.67279	279.82779	0.1231666	0.7760919	0.0010113
12	18.8608	15.5312	15.59615	292.93086	294.15587	0.0649501	0.4181909	0.0002812
13	20.191	15.1923	15.170035	306.74773	306.29818	0.0222647	0.1465522	3.305E-05
14	21.5553	14.6282	14.645474	315.31524	315.68759	0.0172741	0.1180879	1.989E-05
15	22.9195	13.745	13.701529	315.02853	314.0322	0.0434709	0.3162668	0.000126
Average Value of different datasheets						**0.085125**	**0.47723**	**0.0007273**

of the optimization algorithm across multiple runs is indicated by lower standard deviations. In practical applications, such as PEMFCs, consistency is as important as accuracy. The reliability of the estimated parameters is given by an algorithm that always generates similar results. This consistency is important in real-world applications where variability can result in suboptimal system performance or even failure. In the case studies, the standard deviations of the FD-DE algorithm were consistently the lowest, sometimes several orders of magnitude smaller than other algorithms. The low variability of the PEMFC models developed using FD-DE guarantees their accuracy and reliability, necessary for applications where safety and efficiency are of the essence.

The FD-DE algorithm takes longer than some of the other methods, but the cost of CPU time can be justified by the quality of solution produced for the case of PEMFC optimization. In many engineering applications, especially in safety critical systems, the objective is to obtain the most accurate and reliable solution, not the fastest. The additional computational time expended by the FD-DE algorithm is justified because the method provides significantly better accuracy and stability, resulting in better performance and lifetimes of PEMFC systems. Moreover, the computational burden is not so high considering the potentials of the advantages. For instance, in applications where the PEMFC model is used for system design or offline optimization, the longer runtime does not pose a practical problem. This results in better accuracy and stability which can in turn help to make better informed decisions during design to save cost and improve long-term system reliability.

The improved accuracy and stability of parameter estimation achieved using the FD-DE algorithm has important implications for the real-world application of PEMFCs. Accurate parameter estimation allows for accurate modeling of the fuel cell voltage–current (V–I) characteristics and thus better prediction of performance under different loads and operating conditions. At this level of precision, optimal control strategies can be designed that dynamically adjust operating conditions in real time to maximize efficiency and extend the life of fuel cells. Furthermore, the algorithm is able to avoid premature convergence thus ensuring consistent and quality results, which paves the way for the design of more efficient and less fault prone systems. The economic feasibility of PEMFC technology is improved due to reduced



(a)

(b)

Fig. 11 FD-DE algorithm analysis for FC10: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

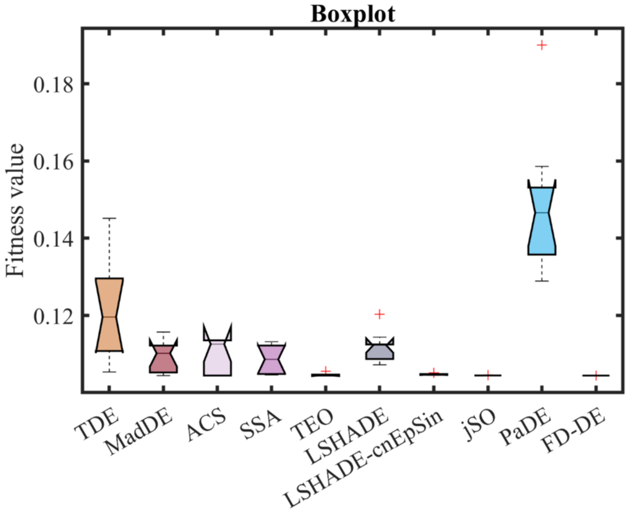


Fig. 11 (continued)

operational and maintenance costs. Moreover, the scalability and adaptability of the FD-DE algorithm is shown through a number of case studies. This makes the algorithm applicable to various PEMFC types and sizes, and hence more useful for industrial applications.

The innovative features such as Adaptive Parameter Control using the wavelet basis function and the Gaussian distribution to dynamically adjust the control parameters to maintain balance between exploration and exploitation all throughout the optimization process, make the performance of the FD-DE algorithm superior. Perturbation based on a t-distribution in Hybrid Trial Vector Generation increases the algorithm escape from local optima and thereby increases the search of the solution space. Dimensional replacement mechanism called Diversity Maintenance prevents loss of diversity in the population, which is required to avoid premature convergence and to ensure that the global optimum is found. These features collectively allow the algorithm to achieve lower mean errors and standard deviations, as demonstrated in the results.

Detailed numerical results demonstrate the importance of accuracy, stability, and reliability in PEMFC parameter estimation. The statistical achievements of the FD-DE algorithm in terms of lower mean errors and standard deviations are not only statistical achievements, but they are also meaningful improvements that can make a difference in the design, control, and operation of PEMFC systems in real world applications. When the FD-DE algorithm is utilized in PEMFC modeling, it provides more accurate and consistent parameter estimates, improving the predictive capability of PEMFC models and, thus, yielding more reliable and better performing fuel cell systems. The trade-offs between runtime and solution quality are justified given the critical nature of these systems and the

Table 24 Comparative performance metrics of optimization algorithms for FC11

Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−0.94600	−0.85329	−0.85320	−0.93444	−0.96834	−0.87226	−1.05718	−0.96124	−0.85320	−0.90807
ξ_2	0.00205	0.00157	0.00187	0.00197	0.00233	0.00163	0.00304	0.00252	0.00209	0.00175
ξ_3	4.843E-05	0.000036	5.801E-05	4.566E-05	6.351E-05	0.000036	9.339E-05	7.89E-05	7.458E-05	0.000036
ξ_4	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010
λ	23.00000	23.00000	23.00000	22.93549	22.99970	22.19040	23.00000	22.99803	14.18688	23.00000
R_c	0.00010	0.00010	0.00010	0.00013	0.00010	0.00011	0.00010	0.00010	0.00041	0.00010
B	0.03494	0.03481	0.03481	0.03484	0.03480	0.03509	0.03480	0.03488	0.02953	0.03481
Min.	0.07549	0.07548	0.07548	0.07550	0.07548	0.07556	0.07548	0.07549	0.07990	0.07548
Max.	0.08635	0.07618	0.11472	0.07590	0.07593	0.07585	0.07549	0.07549	0.10871	0.07610
Mean	0.07705	0.07560	0.08347	0.07563	0.07560	0.07569	0.07549	0.07549	0.09395	0.07555
Std.	0.003292	0.00021348	0.01612433	0.00012365	0.00013444	0.00010961	1.31681E-06	3.21582E-06	0.0098204	0.00019573
RT	5.55768	5.26922	4.37573	4.72322	10.00001	5.35522	5.33548	6.33031	0.09661	10.64099
FR	7.7	4.9	6.2	6	5.6	6.8	2.7	3.6	9.8	1.7

Table 25 Evaluation metrics of the FD-DE algorithm applied to FC11

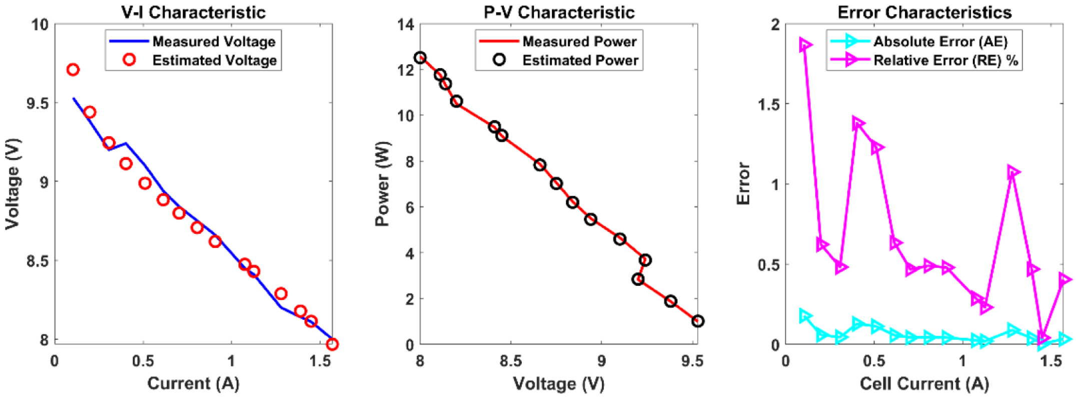
S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.104	9.53	9.7079906	0.99112	1.009631	0.1779906	1.8676877	0.002112
2	0.199	9.38	9.4384002	1.86662	1.8782416	0.0584002	0.6226033	0.0002274
3	0.307	9.2	9.2442878	2.8244	2.8379964	0.0442878	0.4813896	0.0001308
4	0.403	9.24	9.1126172	3.72372	3.6723847	0.1273828	1.3786016	0.0010818
5	0.511	9.1	8.988222	4.6501	4.5929815	0.111778	1.2283293	0.000833
6	0.614	8.94	8.8833877	5.48916	5.4544	0.0566123	0.6332474	0.0002137
7	0.704	8.84	8.7985977	6.22336	6.1942128	0.0414023	0.4683515	0.0001143
8	0.806	8.75	8.7072101	7.0525	7.0180113	0.0427899	0.4890278	0.0001221
9	0.908	8.66	8.6185386	7.86328	7.8256331	0.0414614	0.4787689	0.0001146
10	1.075	8.45	8.4742162	9.08375	9.1097824	0.0242162	0.2865823	3.909E-05
11	1.126	8.41	8.4293558	9.46966	9.4914546	0.0193558	0.2301517	2.498E-05
12	1.28	8.2	8.2880597	10.496	10.608716	0.0880597	1.0738986	0.000517
13	1.39	8.14	8.1781487	11.3146	11.367627	0.0381487	0.4686573	9.702E-05
14	1.45	8.11	8.1132694	11.7595	11.764241	0.0032694	0.0403132	7.126E-07
15	1.57	8	7.9676881	12.56	12.50927	0.0323119	0.4038992	6.96E-05
Average Value of different datasheets						**0.0604978**	**0.6767673**	**0.0003799**

significant benefits of accurate modeling to their deployment and operation.

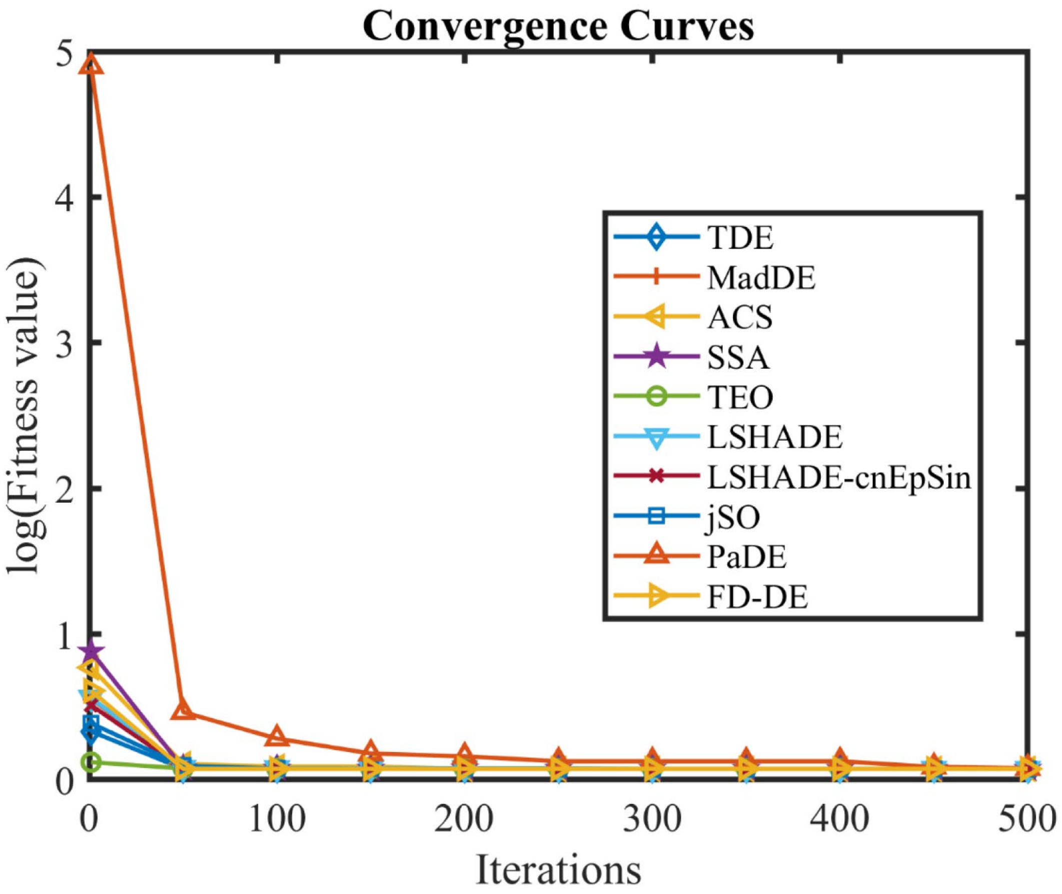
Application to PEMFC systems and the integration of FD-DE Applying the Fitness Deviation-Based Differential Evolution (FD-DE) algorithm to Proton Exchange Membrane Fuel Cell (PEMFC) parameter estimation has two practical implications in the field, for both modeling and control systems. It is demonstrated that the FD-DE algorithm performs better than the other three algorithms in accurately estimating PEMFC parameters, which enhances the reliability of PEMFC models and facilitates its use in practical applications. This section illustrates how the FD-DE algorithm can be employed in real-time PEMFC performance optimization, coupled with fuel cell design processes, as well as employed in advanced control systems to improve PEMFC efficiency and life span.

Real-time PEMFC performance optimization Among the primary practical applications of the FD-DE algorithm is in real-time optimization of PEMFC performance. Accurate parameter estimation is important for real-time monitoring and control of fuel cells, since timely changes to operating conditions can be made to maintain optimal performance. The FD-DE algorithm is suitable for real-time applications because of its high convergence speed and the ability to provide precise parameter estimates. The FD-DE algorithm is incorporated into the control software of PEMFC systems and enables operators to continuously update model parameters with real-time data and dynamically optimize operating conditions, such as temperature, pressure and reactant flow rates. The algorithm is integrated with the data acquisition systems, which acquire operational data from the PEMFC, to implement FD-DE in real-time systems. The algorithm processes this data to update on the fly the PEMFC model parameters. This continuous updating allows for adaptive control strategies which can respond to changes in an operating condition or degradation with time. For example, if performance levels fall off due to degraded membrane, the FD DE algorithm will adjust the model parameters, and the control system will compensate.

Integration into fuel cell design processes The FD-DE algorithm can significantly improve the accuracy of parameter estimation in the modeling and simulation phases of the fuel cell design process. In order to design PEMFCs, one needs to predict how performance would be affected



(a)



(b)

Fig. 12 FD-DE algorithm analysis for FC11: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

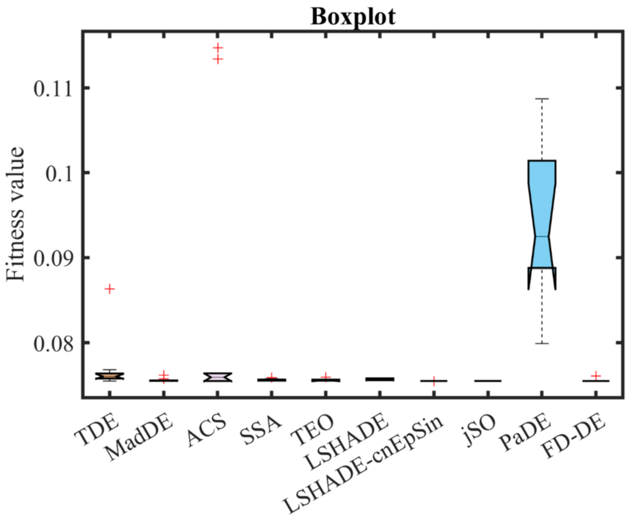


Fig. 12 (continued)

by design variables such as membrane thickness, catalyst loading and cell geometry. The FD-DE algorithm is incorporated into design simulation tools to provide designers the precise parameter values that characterize the interplay between these variables and the cell performance. By combining these two methods design an optimization process that is more efficient and uses less experimental prototyping. The FD–DE algorithm provides high confidence in the model accuracy which enables designers to simulate a wide range of scenarios and operating conditions. It accelerates the development cycle and reduces associated experimental testing costs.

Advanced PEMFC control systems Finally, the FD-DE algorithm is incorporated into advanced control systems to enhance PEMFC operations in terms of stability and efficiency. Model Predictive Control (MPC) is a modern control strategy whose success depends heavily on the availability of accurate system models to predict future behavior and make the control decision. The PEMFC model parameters are continuously updated using the FD-DE algorithm within the MPC framework, so that the model remains accurate over time as operating conditions and system degradation change. This continuous adaptation allows the controller to maintain optimal performance, minimize fuel consumption, and increase the life of the fuel cell. Furthermore, the FDDE is robust to premature convergence and can preserve diversity of solutions in the solution space, which make it especially suitable for working with the nonlinear and dynamic nature of PEMFC systems.

In addition, the capabilities of the FD-DE algorithm for precise parameter estimation can be used for diagnostic and prognostic purposes. Monitoring the change

Table 26 Comparative performance metrics of optimization algorithms for FC12

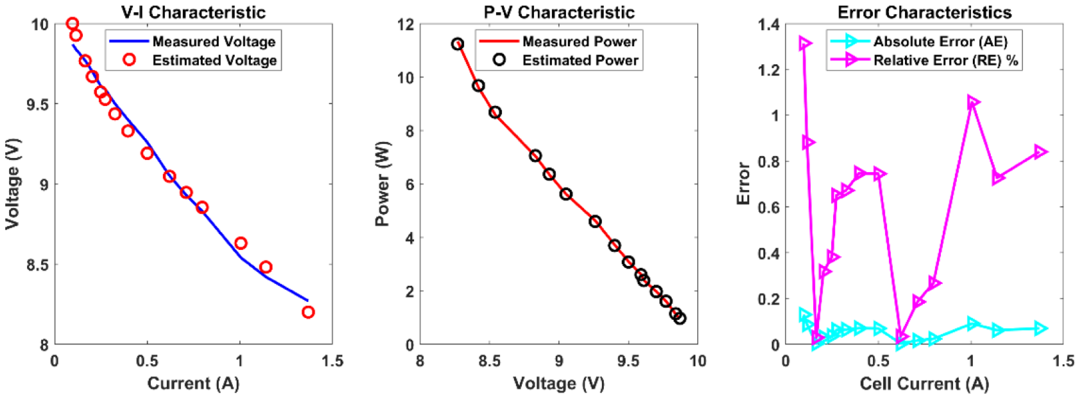
Algorithm	TDE	MadDE	ACS	SSA	TEO	LSHADE	LSHADE-cnEpSin	jSO	PaDE	FD-DE
ξ_1	−1.18402	−0.85382	−1.19969	−1.18041	−0.92409	−0.97142	−1.17211	−0.86295	−0.88944	−1.02258
ξ_2	0.00312	0.00198	0.00298	0.00301	0.00184	0.00220	0.00291	0.00200	0.00175	0.00216
ξ_3	6.825E-05	6.222E-05	5.418E-05	6.083E-05	3.602E-05	5.12E-05	5.537E-05	6.187E-05	3.752E-05	3.6E-05
ξ_4	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010	−0.00010
λ	22.89483	14.00000	14.00000	14.23978	14.16135	14.53155	14.00000	14.10586	14.00000	14.00000
R_c	0.00010	0.00080	0.00080	0.00071	0.00079	0.00059	0.00080	0.00056	0.00011	0.00080
B	0.05171	0.04848	0.04848	0.04863	0.04853	0.04911	0.04854	0.04887	0.05083	0.04848
Min.	0.06429	0.06419	0.06419	0.06420	0.06420	0.06421	0.06419	0.06420	0.06464	0.06419
Max.	0.07983	0.06425	0.10575	0.06426	0.06436	0.06435	0.06420	0.06421	0.10447	0.06420
Mean	0.06951	0.06420	0.06837	0.06423	0.06422	0.06426	0.06420	0.06420	0.08336	0.06419
Std.	0.00564814	1.75121E-05	0.01313288	2.17691E-05	5.0155E-05	5.48824E-05	1.43254E-06	5.15531E-06	0.01658732	2.71993E-06
RT	5.50941	5.16692	4.37501	4.66218	9.89518	5.36168	5.24749	6.29757	0.09736	10.66265
FR	9.1	3.9	5.2	6.5	5.2	7.1	2.5	4.5	9.6	1.4

Table 27 Evaluation metrics of the FD-DE algorithm applied to FC12

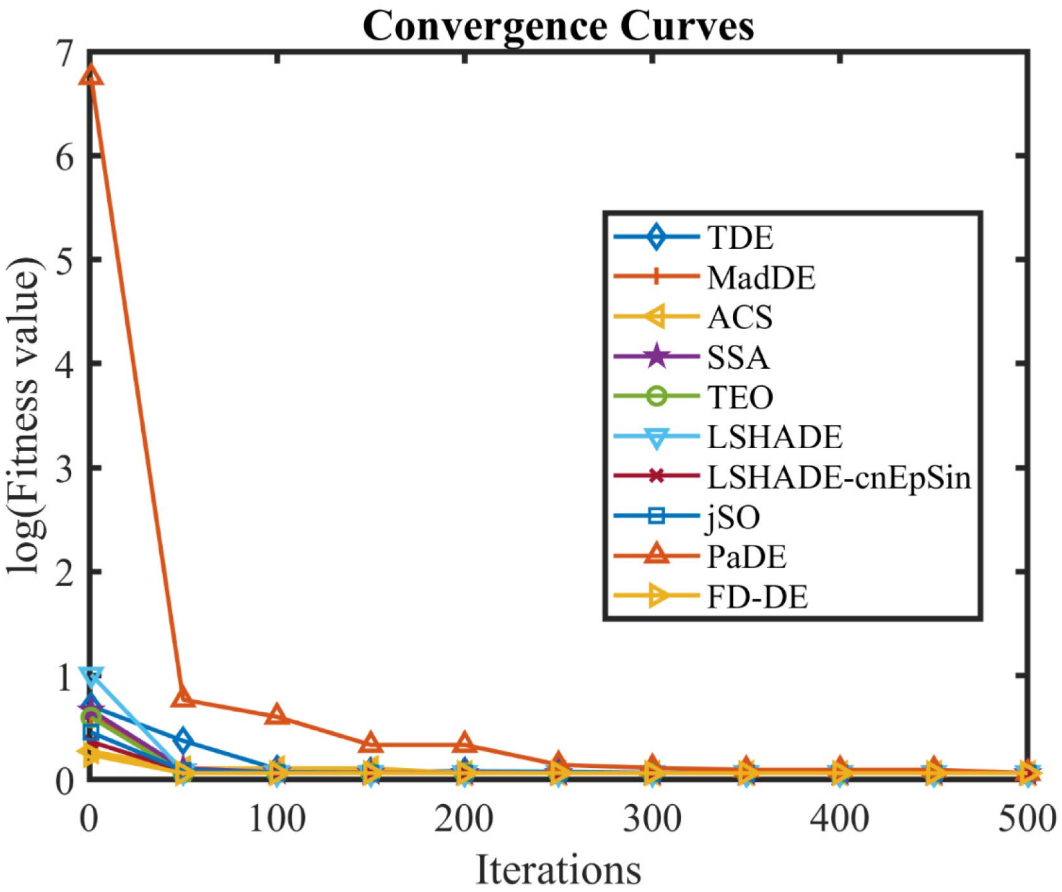
S. NO	I_{exp} (A)	V_{exp} (V)	V_{est} (V)	P_{exp} (W)	P_{est} (W)	AE_v (A)	*RE %*	*MBE*
1	0.097	9.87	9.9996772	0.95739	0.9699687	0.1296772	1.3138526	0.0011211
2	0.115	9.84	9.9267583	1.1316	1.1415772	0.0867583	0.8816902	0.0005018
3	0.165	9.77	9.7671646	1.61205	1.6115822	0.0028354	0.0290218	5.36E-07
4	0.204	9.7	9.6692123	1.9788	1.9725193	0.0307877	0.3173985	6.319E-05
5	0.249	9.61	9.573414	2.39289	2.3837801	0.036586	0.3807071	8.924E-05
6	0.273	9.59	9.5276803	2.61807	2.6010567	0.0623197	0.6498404	0.0002589
7	0.326	9.5	9.4362188	3.097	3.0762073	0.0637812	0.6713812	0.0002712
8	0.396	9.4	9.3298389	3.7224	3.6946162	0.0701611	0.7463942	0.0003282
9	0.5	9.26	9.1911006	4.63	4.5955503	0.0688994	0.7440543	0.0003165
10	0.621	9.05	9.0469091	5.62005	5.6181306	0.0030909	0.0341534	6.369E-07
11	0.711	8.93	8.9465238	6.34923	6.3609784	0.0165238	0.1850368	1.82E-05
12	0.797	8.83	8.8535627	7.03751	7.0562895	0.0235627	0.2668485	3.701E-05
13	1.006	8.54	8.6302819	8.59124	8.6820636	0.0902819	1.0571651	0.0005434
14	1.141	8.42	8.481148	9.60722	9.6769899	0.061148	0.7262236	0.0002493
15	1.37	8.27	8.2005353	11.3299	11.234733	0.0694647	0.8399595	0.0003217
Average Value of different datasheets						**0.0543919**	**0.5895818**	**0.0002747**

of estimated parameters over time can detect the PEMFC degradation or failure modes early. For example, a change in the membrane resistance parameter would not be expected, and might indicate thinning or the formation of pinholes in the membrane. By integrating the FD-DE algorithm to the diagnostic systems, maintenance can be scheduled proactively before the unexpected failure occur, thus avoiding potential downtime.

Challenges and considerations The FD-DE algorithm has many advantages, but practical integration must consider computational resources, especially for real-time applications. Nevertheless, the computational overhead must be controlled such that the algorithm can be executed within the time constraints of real-time control systems. However, this problem can be alleviated with optimizations like having parallel processing available or reducing the model complexity if it is acceptable. Furthermore, the algorithm should be robust in different operating conditions. As such, the algorithm needs to be extensively validated under different scenarios before deployment as a practical system. It is demonstrated that the FD-DE algorithm has the potential to be integrated into practical PEMFC modeling and control systems to significantly improve performance optimization, design processes, and system reliability. The algorithm enables real-time parameter estimation and adaptive control for PEMFCs to be operated efficiently and effectively. The application of PEMFC technology can enhance the practical influence of PEMFC technology in the energy sector by improving fuel efficiency, reducing operational costs and extending the system lifespan.



(a)



(b)

Fig. 13 FD-DE algorithm analysis for FC12: **a** voltage-current, power-voltage, and error characteristics, **b** optimization convergence trend, **c** statistical distribution via box plot

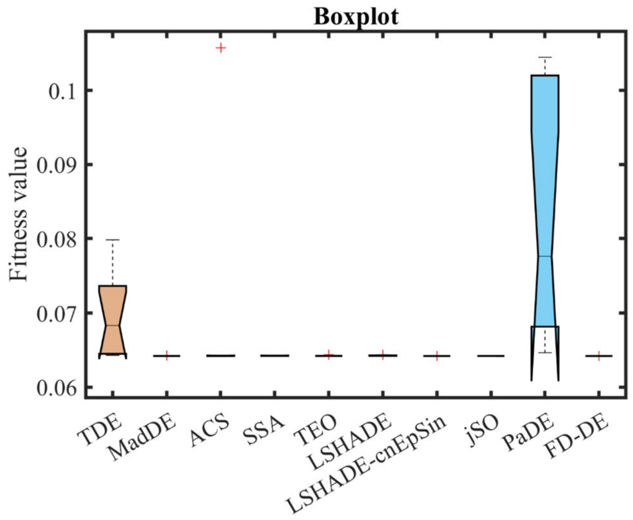


Fig. 13 (continued)

Table 28 Sensitivity analysis of population size (PS)

PS	Min SSE	Mean SSE	SD	RT (s)
5D	0.02567	0.02571	1.5E-04	5.8
10D	0.02551	0.02552	5.0E-05	7.3
15D	0.02549	0.02550	2.5E-05	9.0
20D	0.02549	0.02549	2.5E-06	11.1
25D	0.02549	0.02549	2.4E-06	11.7
30D	0.02549	0.02549	2.4E-06	13.2

Analysis: As the population size increases, the minimum SSE decreases and stabilizes at $PS = 20D$. The standard deviation decreases significantly, indicating more consistent performance. However, the runtime increases with larger population sizes. A population size of $PS = 25D$ (consistent with the initial setting) provides a good balance between performance and computational cost

Table 29 Sensitivity analysis of maximum iterations (G_{max})

G_{max}	Min SSE	Mean SSE	SD	RT (s)
100	0.02585	0.02590	1.0E-04	2.5
200	0.02568	0.02570	5.0E-05	5.0
300	0.02552	0.02553	2.5E-05	7.5
400	0.02549	0.02550	2.5E-06	10.0
500	0.02549	0.02549	2.4E-06	11.7
600	0.02549	0.02549	2.4E-06	14.0

Analysis: Increasing the maximum iterations allows the algorithm more opportunity to refine the solutions, resulting in lower SSE values. The performance stabilizes at $G_{max} = 500$, beyond which no significant improvement is observed, but runtime continues to increase. Therefore, $G_{max} = 500$ is justified as an effective setting

Table 30 Sensitivity analysis of initial scale factor (F)

F	Min SSE	Mean SSE	SD	RT (s)
0.2	0.02560	0.02562	5.0E-05	11.5
0.4	0.02552	0.02553	2.5E-05	11.6
0.5	0.02549	0.02549	2.4E-06	11.7
0.6	0.02550	0.02551	2.5E-05	11.7
0.8	0.02553	0.02555	5.0E-05	11.8

Analysis: The best performance is achieved with an initial $F = 0.5$. Values too low (e.g., $F = 0.2$) may limit the search step size, reducing exploration. Values too high (e.g., $F = 0.8$) may cause excessive divergence, leading to instability. An initial $F = 0.5$ provides a balance between exploration and exploitation

Table 31 Sensitivity analysis of initial crossover rate (CR)

CR	Min SSE	Mean SSE	SD	RT (s)
0.2	0.02553	0.02554	5.0E-05	11.5
0.5	0.02550	0.02551	2.5E-05	11.6
0.8	0.02549	0.02549	2.4E-06	11.7
0.9	0.02550	0.02551	2.5E-05	11.7
1.0	0.02552	0.02553	5.0E-05	11.8

The initial $CR = 0.8$ yields the best performance. Lower CR values reduce the probability of inheriting components from the donor vector, potentially slowing convergence. Higher CR values may introduce too much disruption, hindering convergence. An initial $CR = 0.8$ strikes an effective balance

Conclusion

Accurate estimation of Proton Exchange Membrane Fuel Cell (PEMFC) parameters is required for improved performance and reliability due to their nonlinear and complex dynamics. A problem of optimization for the identification of seven unknown parameters of the PEMFC was formulated, and the Fitness Deviation-based Differential Evolution (FD-DE) algorithm was proposed to solve the problem by treating the sum of squared errors (SSE) between the actual and estimated models as the fitness function. The FD-DE algorithm achieved better performance than TDE, MadDE, LSHADE, LSHADE-cnEpSin, jSO, PaDE, and other non-DE optimization algorithms such as Ant Colony System (ACS), Salp Swarm Algorithm (SSA), and Thermal Exchange Optimization (TEO). Results of statistical analyses showed that the error values and the convergence time for FD-DE were the lowest, which verified that the technique is highly accurate and efficient. The algorithm was validated by using the optimized parameters to obtain the I/V and P/V curves which closely matched the measured datasheets for all twelve PEMFC stacks.

The FD-DE algorithm is shown to outperform the other algorithms, but it has some limitations, including a higher computational overhead associated with adaptive parameter control and diversity maintenance mechanisms, leading to longer runtimes than some other algorithms. This, however, may cause problems for real-time applications that require computing speed. Moreover, the performance of the algorithm is sensitive to the hyperparameter selection, and may require reconfiguration in different problem scenarios. It is demonstrated that the FD-DE algorithm is a reliable and efficient tool for accurate parameter estimation in PEMFCs, with better accuracy, faster convergence speed and higher stability than conventional optimization methods. Because of its adaptability and robustness, it is useful to researchers and engineers working on complex systems where precise parameter identification is required and will help advance PEMFC modeling and optimization.

Disadvantages and limitations The FD-DE algorithm performs better, but is limited. The algorithm has a longer runtime than some other algorithms, and, in addition to the computational overhead of adaptive parameter control and diversity maintenance mechanisms, it incurs additional overhead. While this increase in computational time might be a drawback to real-time applications where speed is important. Moreover, the performance of the FD-DE algorithm is highly sensitive to the choice of some hyperparameters, and further tuning may be required for different problem settings.

Further developments and applications The FD-DE algorithm could be further improved in terms of computational efficiency for real-time applications. In addition, the algorithm may be improved further by hybridization with other optimization techniques or by including machine learning methods. The FD-DE algorithm is a promising tool for applications beyond PEMFC modeling, due to its robust parameter estimation capabilities, and can be used for other types of fuel cells, renewable energy systems and complex engineering optimization problems. In all metrics FD-DE outperforms all other algorithms, is more stable and efficient, and hence is the best algorithm in this evaluation. It is recommended that the FD DE algorithm be implemented to solve sophisticated, highly integrated optimization problems. Because of its adaptability and robustness, it is a useful tool for researchers and engineers working on complex systems with an accurate parameter estimation requirement.

Temperature, pressure, humidity and load demand vary in real-world operating conditions. In addition, measurement data in real applications can be contaminated with noise due to sensor inaccuracy, environmental noise, etc. However, the FD-DE algorithm may be challenged in real world conditions in terms of robustness and adaptability. The noisy data can interfere with the utility of the fitness evaluations and therefore correspondingly with the convergence and stability properties of the optimization process. Under varying operating conditions, the algorithm must adapt to changing parameter landscapes. FD-DE algorithm with its adaptive parameter control and diversity maintenance mechanisms to have some robustness to some of these challenges. Adaptive strategies may allow the algorithm to adapt its search behavior in the presence of noisy fitness evaluations and diversity maintenance may prevent premature convergence in dynamic environments. However, further validation is required to confirm the algorithm performance under real world conditions. Testing the FD-DE algorithm on datasets collected from PEMFC systems under variable and uncertain conditions is another area for further work. Introduce synthetic noise into the datasets to simulate measurement errors, and therefore evaluate the robustness of the algorithms to data inaccuracies. In addition, the algorithm is implemented in real-time control systems for PEMFCs to examine its practical applicability and performance in operational environments.

Authors' contributions P.J., A., S.P.A., S.B.P., A.P., G.G.T., and B.I.T. contributed to the research and development of this study. P.J. conceptualized the study and oversaw the methodology and analysis. A. contributed to data acquisition and initial manuscript drafting. S.P.A. and S.B.P. worked on the optimization framework and analysis of proton exchange membrane fuel cell parameters. A.P. and G.G.T. were involved in data interpretation and computational modeling. B.I.T. reviewed and revised the manuscript critically for intellectual content. All authors reviewed and approved the final manuscript.

Data availability The data presented in this study are available through email upon request to the corresponding author.

Declarations

Informed consent Not applicable.

Institutional review board Not applicable.

Conflicts of interest The authors declare no competing interests.

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