RESEARCH ARTICLE

An Intelligent Decision Support System for Selecting Optimal AI-Powered Assistive Technology for Individuals with Disabilities

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Abstract
We introduce a novel extension of conventional fuzzy sets in this paper: called trimorphic fuzzy sets. As per our research, trimorphic fuzzy sets which exhibit greater capability than intuitionistic fuzzy sets, picture fuzzy sets and bipolar fuzzy sets, present a viable approach to address ambiguity and uncertainty in decision-making scenarios. We present a complete characterization of trimorphic fuzzy sets, discuss their properties, and consider applications to real-world decision-making scenarios. We also present a case study to further highlight the practical applications of trimorphic fuzzy sets. We look into a few aggregation strategies for trimorphic fuzzy data in this work. We create the MCDM method using trimorphic fuzzy aggregation operators to help people with disabilities choose AI-Powered Assistive Technologies. We have also presented the extended TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method with trimorphic fuzzy numbers. A numerical example for selection of AI-Powered Assistive Technologies using TOPSIS method is also provided.

Keywords Fuzzy set · Trimorphic fuzzy sets · Decision-making · Aggregation operators · Disabilities · Assistive technologies · TOPSIS method

1 Introduction

A fuzzy set is a type of mathematical structure that is an extension of the classical set [32]. There is a degree of membership connected to every element in the fuzzy set. The value of this membership degree ranges from 0 to 1. Numerous domains, such as artificial intelligence, control systems, and decision-making, find extensive use for fuzzy sets [35]. In an expansion of the fuzzy set, Atanassov presented the intuitionistic fuzzy set [1]. For this type of fuzzy set, there exists a relationship between the degree of membership and the degree of non-membership. Various problems with accuracy and uncertainty in decision-making processes have been resolved with this type of fuzzy set. Cuong [7] introduced a picture fuzzy set in 2013 which was an additional variation of the fuzzy set. This kind of fuzzy set whose elements are linked to three values (membership, non-membership and neutrality) is better equipped to handle issues of ambiguity and imprecision during the decision-making process. However, a novel kind of fuzzy set known as bipolar fuzzy set (BFS) [8] has emerged the elements of which are correlated with the degree of both positive and negative memberships. Picture fuzzy and other fuzzy sets that have already

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been discussed however are not always applicable. The situations that require consideration of one positive and two negative memberships cannot be discussed by all the existing types of fuzzy set.

In order to solve this, we have created a novel class of fuzzy sets called trimorphic fuzzy sets (TrFS). Better than intuitionistic fuzzy sets, picture fuzzy sets and BFSs [1, 2, 25, 34], these sets build upon the foundations of classical fuzzy sets. Three values are assigned to each element in a TrFS: one positive membership function and two negative membership functions. Because of this, TrFS is more adaptable and complete than other kinds of fuzzy sets. For instance, mobility is a positive (good) when selecting a wheelchair for a disabled person while increased social barriers or reliance on caregivers are negative (bad). An expert assigned a positive value of 0.6 and two negative values of -0.4 each. A set $(0.6 - 0.4 - 0.4)$ is thus produced. TrFS is highly capable of handling this type of data something that other fuzzy sets are unable to do. In addition, the TrFS sets are described using three membership grades $\widehat{\rho t}_{\check{D}}$, $\widehat{\beta t}_{\check{D}}$ and $\widehat{\gamma t}_{\check{D}}$ the sum of absolute which is at most 1, i.e., $0 \leq \left|\widehat{\rho t}_{\check{D}} + \widehat{\beta t}_{\check{D}} + \widehat{\gamma t}_{\check{D}}\right| \leq 1$.

Consider the following example to illustrate more the idea of an TrFS set: an expert expresses a preference for attribute values with positive membership 0.4, one negative membership -0.3, and another negative membership -0.2. Then clearly $0.4 + (-0.3) + (-0.4) = -0.3$. Consequently the PF, BF or all other types of fuzzy set are unable to handle this kind of data. However, we have the TrFS set. $|0.4 + (-0.3) + (-0.4)| = |-0.3| = 0.3 \leq 1$, TrFS sets can therefore deal with such situations quite well.

Fuzzy multi-criteria decision-making (MCDM) is one method [32] designed to manage uncertainty in data. In contemporary decision science, MADM serves as a valuable research tool. The selection and the use of appropriate MCDM techniques to solve complex problems are one of the difficult decisions that decision-makers must make. MCDM techniques have been successfully applied to handle these issues, for example, in the fields of artificial intelligence, information technology, farming, medicine, business, and trade. The integration of MCDM techniques, which have been successfully applied in a variety of application areas, is the subject of this study. For instance, making the best decision is the aim of this strategy. Management science has made extensive use of MADM [14], economics [22], and operation research [21] applications that incorporate attribute values that are both qualitative and quantitative. In life, there are a lot of imprecise facts that must be considered when making decisions. Experts frequently find it difficult to articulate their opinions with precise values in decision-making scenarios and to pinpoint the precise values of potential Alts when faced with competing criteria or attributes [4, 28, 29].

Disability is a social construct influenced by cultural norms environmental variables and the availability of services and infrastructure, all of which should not be overlooked. In order to minimize the effects of disabilities and promote greater equity and inclusion for people with diverse abilities, efforts must be made to create inclusive environments and promote accessibility [20]. Utilizing developments in robotics artificial intelligence and sensor technologies, these assistive technologies (ATs) are always changing and offering more individualized and practical solutions for a wide range of disability categories [5]. When it comes to reducing barriers that individuals with disabilities must face in order to participate more fully in a range of activities, creative solutions known as assistive technologies or ATs are developed [31]. The combination of artificial intelligence (AI) and ATs enhances the quality of life for individuals with disabilities [13]. For instance, speech recognition technology makes it easier for those who struggle with speech to communicate by utilizing natural language processing [3, 33]. Computer vision systems assist blind individuals with object recognition and environment description through sophisticated algorithms making navigation easier [26]. Individuals with limited mobility can interact with electronics more easily using hand gestures [18]. In addition, the integration of AI-powered personalized recommendations and smart home systems promotes independence and accessibility within the home. Furthermore, improved mobility aids and prosthetics with artificial intelligence offer more natural movement and control. The quality of life for those with disabilities is eventually improved by these technologies which address a variety of disability categories and provide specialized assistance to address specific problems. People with social communication challenges can benefit from emotion recognition technology and those with cognitive impairments can benefit from cognitive assistance technologies [24]. An array of learning requirements is also satisfied by AI-powered educational materials, provided that privacy accessibility and usability are given high priority during the development and implementation process. The integration of AI into ATs generally has great potential to improve quality of life and promote inclusively independence. In recent years, the field of assistive technology has advanced significantly particularly with the introduction of intelligent computer programs and sophisticated decision-making tools [10]. The data can be processed and a decision made using AI techniques.

Trimorphic fuzzy sets can be applied to resolve a variety of imprecise and uncertain decision-making problems. Multi-criteria decision analysis can make use of the trimorphic fuzzy set. We have developed a number of aggregation operators with trimorphic fuzzy information.

Flaws of using Existing Aggregation Operators

1. Struggling with high uncertainty, leading to inaccurate results.
2. Understanding to extreme values.
3. Troubling in interpretation of outcomes.
4. Partial litheness in correcting aggregation procedure.

Despite the fact that IFS or PFS is successfully applied to real-world problems, but certain real-world situations are beyond the scope of these theories. The flavor of food is a prime example [30]. Foods with sweet flavors have a positive membership value whereas those with bitter tastes have a negative one. Foods with different flavors like chilly or acidic stand for neutral membership values. However, traditional generalizations of fuzzy sets are binary in nature, assigning a degree of membership to an element and its complement. In many decision-making problems, it is often necessary to express some reluctance or uncertainty regarding the extent of membership. The addition of a third value by a trimorphic fuzzy set allows for the explicit representation of hesitancy or neutrality and provides a more detailed structure. TrFS which is a generalized version of BFS and IFS. FS, IFS and BFS data are all contained in TrFS: a hybrid structure. But these trimorphic fuzzy numbers (TrFNs) cannot be combined using any of the previously listed techniques. As such compiling these, TrFNs is a challenging task. We will use traditional arithmetic and geometry operations as a basis to create several trimorphic fuzzy aggregation processes in this study in order to address these issues. We develop trimorphic fuzzy averaging and trimorphic fuzzy geometric aggregation operators.

Solutions to the flaws using Tripolar Fuzzy Aggregation Operators

1. Generates the results which are more accurate.
2. Proposes exhaustive and fathomable outcomes to facilitate better decision-making.
3. Offers superior liberty to alter the technique to meet distinctive necessities.
4. More effectively seizures vagueness, vagueness, uncertainty and imprecision.
5. Copes complicated relationships among the criteria.

A major source of motivation for us in developing our current work was decision-making difficulties in aggregation systems with imprecise information. This article's main goal is to illustrate these aggregation operations under trimorphic fuzzy data for assessing the various decision-priority options throughout the MCDM process. Apart from the noteworthy and complicated techniques that have been previously developed in this domain, we have taken great care to ensure that our suggested procedures are fully tested and capable of resolving any existing issues while also addressing real-world concerns, provide MCDM formulations based on trimorphic fuzzy data and a brief explanation of the decision-making procedure using established operators, and analyze a case to the assess the effectiveness of various AI-powered cognitive assistive technologies for people with disabilities to show the viability and usefulness of the newly suggested methods.

The idea of TOPSIS method was established by Hwang and Yoon [9]. Many authors developed this method later. The high flexibility of the TOPSIS method allows us to add additional extensions in order to make the best choices in different situations. Practically, to solve many theoretical and real-world problems, TOPSIS and its modifications are used [15, 19]. The results can be easily evaluated using TOPSIS method in complex decision-making, which contains a lot of qualitative information. For ranking and selection of Altrs, the TOPSIS method is a useful and practical technique. In this paper, we describe the concept of the TOPSIS algorithm for trimorphic fuzzy data. An extension of the trimorphic fuzzy TOPSIS technique to a group decision environment is also investigated.

In this innovative approach, the trimorphic fuzzy sets to the conventional fuzzy set is meant to handle situations where multiple complex instructions are provided. For example, the selection of AI-powered assistive technologies for people with disabilities and employment advancement, etc. To sum up, by tackling situations where limitations exist like job selection and promotion, the designed trimorphic fuzzy set makes a significant contribution to the field of fuzzy sets. The integration of AI into assistive technologies holds great promise for enhancing quality of life independence and transparency as long as privacy accessibility and usability are prioritized throughout the development and implementation phases.

The current paper is formatted in this manner. In Sect. 2, the fundamental ideas and fundamental operational guidelines of the fuzzy sets are briefly covered. In Sect. 3, several aggregation operators for trimorphic fuzzy data are discussed. Both a trimorphic fuzzy weighted geometric (TrFWG) operator and a trimorphic fuzzy weighted arithmetic (TrFWA) operator are developed. We created a model using the proposed operators to handle multiple attribute decision-making issues in a trimorphic fuzzy environment in Sect. 4. In Sect. 5, we use the MADM technique for selecting AI-Powered assistive technology for individuals with disabilities. The suggested aggregation operators under trimorphic information are compared to other existing aggregation operators. An extended TOPSIS approach with trimorphic fuzzy numbers is described in Sect. 6. We use the TOPSIS technique for selecting AI-Powered assistive technology for individuals with disabilities. A comparison analysis of TOPSIS method with other existing techniques is also provided in Sect. 6.2. Section 7 concludes the paper with a few closing thoughts.

2 Preliminaries

In this section, we have demonstrated a number of basic definitions that will influence the framework for the discussions and analyses that are presented later in this paper. These definitions are essential for outlining a standard understanding of the main concepts and language. Our goal in defining these terms clearly is to ensure precision and clarity in our discussions and paper. As we delve further into the topic, it is imperative that you have a firm grasp of these fundamental definitions in order to engage in meaningful and thoughtful dialog. This section serves as an introduction to the next sections which will apply and analyze these concepts in greater detail.

2.1 Fuzzy Set

A fuzzy set is a set of numbers that covers a range of items with varying enrollment grades. Unlike traditional sets in which an item is either included in the set or not, a fuzzy set takes partial enrollment into account allowing an item to have a partially defined place within the set. This concept is particularly useful in domains where vulnerability and ambiguity are common, such as artificial intelligence decision-making and pattern recognition. Fuzzy sets are widely used to illustrate and make sense of ambiguous or loose data providing a more flexible and logical framework for handling confusing real-world data. Fuzzy sets are a crucial tool in various domains of logical and design inquiry because they take into account the representation of consistent variations between enrollment and non-participation enabling a more nuanced understanding of suspicious or unclear peculiarities.

Definition 2.1 [32] Let $\overline{L}$ be a fixed set. A fuzzy set $\check{D}$ of $\overline{L}$ is given by

$$\check{D} = \{(\bar{l}, \widehat{\rho t}_{\check{D}}(\bar{l}))|\bar{l} \in \overline{L}\},$$

where $\widehat{\rho t}_{\check{D}}(\bar{l}) : \overline{L} \to [0, 1]$ is the membership function of a fuzzy set $\check{D}$.

Shortcoming:

Cannot model hesitation, opposition, or neutrality explicitly.

Restricted to single-dimension decision-making.

Definition 2.2 [32] Let $\check{D}_1 = \{(\bar{l}, \widehat{\rho t}_{\check{D}_1}(\bar{l}))|\bar{l} \in \overline{L}\}$ and $\check{D}_2 = \{(\bar{l}, \widehat{\rho t}_{\check{D}_2}(\bar{l}))|\bar{l} \in \overline{L}\}$ be two FSs, and $\tilde{n} > 0$, then,

1. $\check{D}_2 \subset \check{D}_1$ iff $\widehat{\rho t}_{\check{D}_2}(\bar{l}) < \widehat{\rho t}_{\check{D}_1}(\bar{l})$;
2. $\check{D}_2 = \check{D}_1$ iff $\widehat{\rho t}_{\check{D}_2}(\bar{l}) = \widehat{\rho t}_{\check{D}_1}(\bar{l})$;
3. $\check{D}^c = 1 - \widehat{\rho t}_{\check{D}}(\bar{l})$;
4. $\check{D}_2 \cup \check{D}_1 = \max\left\{\widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l})\right\}$;
5. $\check{D}_2 \cap \check{D}_1 = \min\left\{\widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l})\right\}$;
6. $\check{D}_2 \oplus \check{D}_1 = \left\{\widehat{\rho t}_{\check{D}_2}(\bar{l}) + \widehat{\rho t}_{\check{D}_1}(\bar{l}) - \widehat{\rho t}_{\check{D}_2}(\bar{l})\widehat{\rho t}_{\check{D}_1}(\bar{l})\right\}$;
7. $\check{D}_2 \otimes \check{D}_1 = \left\{\widehat{\rho t}_{\check{D}_2}(\bar{l}) \cdot \widehat{\rho t}_{\check{D}_1}(\bar{l})\right\}$;
8. $\tilde{n}\check{D} = \{1 - (1 - \widehat{\rho t}_{\check{D}}(\bar{l}))^{\tilde{n}}\}$;
9. $\check{D}^{\tilde{n}} = \{(\widehat{\rho t}_{\check{D}}(\bar{l}))^{\tilde{n}}\}$;
10. $\tilde{n}(\check{D}_1 \oplus \check{D}_2) = \tilde{n}\check{D}_1 \oplus \tilde{n}\check{D}_2$;
11. $(\tilde{n}_1 \oplus \tilde{n}_2)\check{D} = \tilde{n}_1\check{D} + \tilde{n}_2\check{D}$.

2.2 Intuitionistic Fuzzy Set

An expansion of the fuzzy set that takes into account how vulnerability and delay are portrayed in dynamic cycles is called the Intuitionistic fuzzy set (IFS). Every element in uncertainty is reduced to a level of non-enrollment and faltering in addition to a level of participation in a set. This takes into account presenting data in a more nuanced manner particularly when the boundaries between classifications are blurry. The concept of uncertainty has been applied to a variety of domains including picture-processing control frameworks emotionally supportive network selection and design acknowledgment. A more flexible method for handling loose and unsure data is provided by uncertainties which incorporates the idea of faltering. Enhancements to intuitionistic fuzzy sets have contributed to the development of fuzzy reasoning and its uses providing a more comprehensive and useful approach to handling ambiguity and vulnerability when making decisions.

Definition 2.3 [1] Let $\overline{L}$ be a fixed set. An intuitionistic fuzzy set $\check{D}$ of $\overline{L}$ is defined as

$$\check{D} = \{(\bar{l}, \widehat{\rho t}_{\check{D}}(\bar{l}), \widehat{\beta t}_{\check{D}}(\bar{l})|\bar{l} \in \overline{L}\},$$

where for each element $\bar{l} \in \overline{L}$, the membership function is $\widehat{\rho t}_{\check{D}}(\bar{l}) : \overline{L} \to [0, 1]$ and the non-membership function is $\widehat{\beta t}_{\check{D}}(\bar{l}) : \overline{L} \to [0, 1]$, with $0 \leq \widehat{\beta t}_{\check{D}}(\bar{l}) + \widehat{\rho t}_{\check{D}}(\bar{l}) \leq 1$ for every $\bar{l} \in \overline{L}$.

Shortcoming:
Cannot differentiate between conflicting positive and negative sentiments.
No explicit neutrality component.

Definition 2.4 [1] The two IFS are $\check{D}_2 = \left(\widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\beta t}_{\check{D}_2}(\bar{l})\right)$ and $\check{D}_1 = \left(\widehat{\rho t}_{\check{D}_1}(\bar{l}), \widehat{\beta t}_{\check{D}_1}(\bar{l})\right)$. Yager and Xu established operations on these sets as follows:

1. $\check{D}_2 \subset \check{D}_1$ iff $\widehat{\rho t}_{\check{D}_2}(\bar{l}) < \widehat{\rho t}_{\check{D}_1}(\bar{l})$ and $\widehat{\beta t}_{\check{D}_2}(\bar{l}) > \widehat{\beta t}_{\check{D}_1}(\bar{l})$;
2. $\check{D}^c = (\widehat{\beta t}_{\check{D}}(\bar{l}), \widehat{\rho t}_{\check{D}}(\bar{l}))$
3. $\check{D}_2 \cup \check{D}_1 = \left(\max\left\{\widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l})\right\}, \min\left\{\widehat{\beta t}_{\check{D}_2}(\bar{l}), \widehat{\beta t}_{\check{D}_1}(\bar{l})\right\}\right)$;

4. $\check{D}_2 \oplus \check{D}_1 = \left\{ \widehat{\rho t}_{\check{D}_2}(\bar{l}) + \widehat{\rho t}_{\check{D}_1}(\bar{l}) - \widehat{\rho t}_{\check{D}_2}(\bar{l})\widehat{\rho t}_{\check{D}_1}(\bar{l}), \widehat{\beta t}_{\check{D}_2}(g)\, \widehat{\beta t}_{\check{D}_1}(g) \right\}$;
5. $\check{D}_2 \otimes \check{D}_1 = \left\{ \widehat{\rho t}_{\check{D}_2}(\bar{l})\widehat{\rho t}_{\check{D}_1}(\bar{l}), \widehat{\beta t}_{\check{D}_2}(\bar{l}) + \widehat{\beta t}_{\check{D}_1}(\bar{l}) - \widehat{\beta t}_{\check{D}_2}(\bar{l})\widehat{\beta t}_{\check{D}_1}(\bar{l}) \right\}$;
6. $\check{D}_2 \cap \check{D}_1 = \left(\min\left\{ \widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l}) \right\}, \max\left\{ \widehat{\beta t}_{\check{D}_2}(\bar{l}), \widehat{\beta t}_{\check{D}_1}(\bar{l}) \right\} \right)$;
7. $\tilde{n} A \check{D}_1 = \{1 - (1 - \widehat{\rho t}_{\check{D}}(\bar{l}))^{\tilde{n}}, (\widehat{\beta t}_{\check{D}}(\bar{l}))^{\tilde{n}}\}$;
8. $\check{D}^{\tilde{n}} = \{(\widehat{\rho t}_{\check{D}}(\bar{l}))^{\tilde{n}}, 1 - (1 - \widehat{\beta t}_{\check{D}}(\bar{l}))^{\tilde{n}}\}$;
9. $\tilde{n}(\check{D}_1 \oplus \check{D}_2) = \tilde{n}\check{D}_1 \oplus \tilde{n}\check{D}_2$;
10. $(\tilde{n}_1 \oplus \tilde{n}_2)\check{D} = \tilde{n}_1\check{D} + \tilde{n}_2\check{D}$.

2.3 Bipolar Fuzzy Set

Expanding upon traditional fuzzy sets, bipolar fuzzy sets take into account a more nuanced representation of vulnerability and equivocality. Bipolar fuzzy sets incorporate the notions of non-participation and against enrollment in addition to the participation and non-enrollment considered in traditional fuzzy sets. It is especially useful in applications where precise boundaries are not always evident because this extra feature allows for a more comprehensive demonstration of vulnerability. In this direction, certain certifiable situations have an inherent vulnerability that bipolar fuzzy sets can capture. Leaders can more effectively address their vulnerability regarding the various options available by advocating against participation and non-enrollment. This can lead to more intelligent and robust dynamic cycles particularly in perplexing and questionable circumstances. Bipolar fuzzy sets provide a more flexible and adaptable method of expressing the inherent uncertainty in information when it comes to design acknowledgment. Conventional fuzzy sets may find it difficult to capture the subtleties of unclear or ambiguous examples, but bipolar fuzzy sets provide a more comprehensive representation that is more likely to align with the confusing concept of real information.

Definition 2.5 [8] Let $\overline{L}$ be fixed set. A bipolar fuzzy set $\check{D}$ of $\overline{L}$ is defined as,

$$\check{D} = \{(\bar{l}, \widehat{\rho t}_{\check{D}}(\bar{l}), \widehat{\gamma t}_{\check{D}}(\bar{l}) | \bar{l} \in \overline{L}\},$$

where for each element $\bar{l} \in \overline{L}$, membership function is $\widehat{\rho t}_{\check{D}}(\bar{l}) : \overline{L} \to [0, 1]$ and $\widehat{\gamma t}_{\check{D}}(\bar{l}) : \overline{L} \to [-1, 0]$ is negative membership function respectively, with $-1 \leq \widehat{\rho t}_{\check{D}}(\bar{l}) + \widehat{\gamma t}_{\check{D}}(\bar{l}) \leq 1$ here $\pi_{\check{D}}(\bar{l}) = 1 - \widehat{\rho t}_{\check{D}}(\bar{l}) + \widehat{\gamma t}_{\check{D}}(\bar{l})$, $\pi_{\check{D}}(\bar{l})$ is hesitancy.

Shortcoming:

Cannot represent neutrality or hesitation explicitly.

Restricted to positive and negative perspectives. Cannot represent a situations that require one positive and two negative memberships.

Definition 2.6 [8] Let $\check{D}_1 = \left(\widehat{\rho t}_{\check{D}_1}(\bar{l}), \widehat{\gamma t}_{\check{D}_1}(\bar{l}) \right)$ and $\check{D}_2 = \left(\widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\gamma t}_{\check{D}_2}(\bar{l}) \right)$ be two BFSs, and $\tilde{n} > 0$, then

1. $\check{D}_2 \subset \check{D}_1$ iff $\widehat{\rho t}_{\check{D}_2}(\bar{l}) < \widehat{\rho t}_{\check{D}_1}(\bar{l})$ and $\widehat{\gamma t}_{\check{D}_2}(\bar{l}) > \widehat{\gamma t}_{\check{D}_1}(\bar{l})$;
2. $\check{D}_1 \cap \check{D}_2 = \left(\min\left\{ \widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l}) \right\}, \max\left\{ \widehat{\gamma t}_{\check{D}_2}(\bar{l}), \widehat{\gamma t}_{\check{D}_1}(\bar{l}) \right\} \right)$;
3. $\check{D}_1 \cup \check{D}_2 = \left(\max\left\{ \widehat{\rho t}_{\check{D}_2}(\bar{l}), \widehat{\rho t}_{\check{D}_1}(\bar{l}) \right\}, \min\left\{ \widehat{\gamma t}_{\check{D}_2}(\bar{l}), \widehat{\gamma t}_{\check{D}_1}(\bar{l}) \right\} \right)$;
4. $\check{D}^c = \left(1 - \widehat{\rho t}_{\check{D}}(\bar{l}), \left|\widehat{\rho t}_{\check{D}}(\bar{l})\right| - 1\right)$.

Definition 2.7 [8] Let $\check{D} = (\widehat{\rho t}_{\check{D}}, \widehat{\gamma t}_{\check{D}})$ be bipolar fuzzy number, then the score function of $\check{D}$ is defined as

$$S(\check{D}) = \frac{1}{2}(1 + \widehat{\rho t}_{\check{D}} + \widehat{\gamma t}_{\check{D}}), \qquad S(\check{D}) \in [0, 1]$$

The accuracy function H of $\check{D}$ is formulated as

$$H(\check{D}) = \frac{1}{2}(\widehat{\rho t}_{\check{D}} - \widehat{\gamma t}_{\check{D}}), \overline{L}(\check{D}) \in [0, 1]$$

If $S(\check{D}_1) \leq S(\check{D}_2)$ or $S(\check{D}_1) = S(\check{D}_2)$ but $H(\check{D}_1) \leq H(\check{D}_2)$, then $\check{D}_1 \prec \check{D}_2$ If $S(\check{D}_1) = S(\check{D}_2)$ and $H(\check{D}_1) = H(\check{D}_2)$ then $\check{D}_1 = \check{D}_2$.

Definition 2.8 [23] Let us suppose a universal set X. A tripolar fuzzy set $\ddot{E}$ is an object having the form,

$$\ddot{E} = \{x, \urcorner_{\ddot{E}}(x), \varsigma_{\ddot{E}}(x), \ddot{e}_{\ddot{E}}(x)\},$$

here $\urcorner_{\ddot{E}} : X \to [0, 1]$, represents positive membership function, $\varsigma_{\ddot{E}} : X \to [0, 1]$ represents the negative membership function TFS and $\ddot{e}_{\ddot{E}} : X \to [-1, 0]$ represents the irrelevant membership function. For simplicity, $\ddot{E} = (\urcorner_{\ddot{E}}, \varsigma_{\ddot{E}}, \ddot{e}_{\ddot{E}})$ has been used instead of $\ddot{E} = \{x, (\urcorner_{\ddot{E}}(x), \varsigma_{\ddot{E}}(x), \ddot{e}_{\ddot{E}}(x))\}$. Also $-1 \leq \urcorner_{\ddot{E}} + \varsigma_{\ddot{E}} + \ddot{e}_{\ddot{E}} \leq 1$.

3 Trimorphic Fuzzy Set

In this section, a novel extension of the fuzzy set, a Trimorphic Fuzzy Set, is introduced. It is used where a function has one positive value and two negative values, for example, to select a wheelchair for disabled people. Mobility is a positive value and increases social barriers, and dependence on caregivers is a negative value. The application of TrFSs in the process of the selection of the wheelchairs for individuals with disabilities demonstrates its capacity to address real-world problems.

Importance and Implications

Situations that require consideration of one positive and two negative memberships are better suited for trimorphic fuzzy sets. In sentiment analysis, decision-making processes involving opposing viewpoints and multi-criteria decision analysis (MCDA) where various alternatives may have one positive and two negative aspects are frequently employed.

Definition 3.1 A Trimorphic fuzzy set is an object having the form

$$\check{D} = \{\bar{l}, (\widehat{\rho t}_{\check{D}}(\bar{l}), \widehat{\beta t}_{\check{D}}(\bar{l}), \widehat{\gamma t}_{\check{D}}(\bar{l}))\}$$

here $\widehat{\rho t}_{\check{D}} : \overline{L} \to [0, 1]$, represents a positive degree of membership and $\widehat{\beta t}_{\check{D}} : \overline{L} \to [-1, 0]$ and $\widehat{\gamma t}_{\check{D}} : \overline{L} \to [-1, 0]$ represents a negative degree of membership a counter property of Trimorphic Fuzzy Sets $\check{D}$. For simplicity, $\check{D} = (\widehat{\rho t}_{\check{D}}, \widehat{\beta t}_{\check{D}}, \widehat{\gamma t}_{\check{D}})$ has been used instead of $\check{D} = \{\bar{l}, (\widehat{\rho t}_{\check{D}}(\bar{l}), \vec{\lambda} s_{\check{D}}(\bar{l}), \widehat{\gamma t}_{\check{D}}(\bar{l}))\}$.Also $0 \leq \left|\widehat{\rho t}_{\check{D}} + \widehat{\beta t}_{\check{D}} + \widehat{\gamma t}_{\check{D}}\right| \leq 1$ Let $\check{D} = (\widehat{\rho t}_{\check{D}}, \widehat{\beta t}_{\check{D}}, \widehat{\gamma t}_{\check{D}})$. be a Trimorphic fuzzy number (TrFN). We define an accuracy function and a score function for now.

Definition 3.2 Let $\check{G} = (\widehat{\rho t}_{\check{G}}, \widehat{\beta t}_{\check{G}}, \widehat{\gamma t}_{\check{G}})$ and $\check{D} = (\widehat{\rho t}_{\check{D}}, \widehat{\beta t}_{\check{D}}, \widehat{\gamma t}_{\check{D}})$ be two trimorphic fuzzy numbers and $\tilde{n} \in [0, 1]$, then

1. $\check{G} \wedge \check{D} = (\min(\widehat{\rho t}_{\check{G}}, \widehat{\rho t}_{\check{D}}), \max(\widehat{\beta t}_{\check{G}}, \widehat{\beta t}_{\check{D}}), \max(\widehat{\gamma t}_{\check{G}}, \widehat{\gamma t}_{\check{D}}))$
2. $\check{G} \vee \check{D} = (\max(\widehat{\rho t}_{\check{G}}, \widehat{\rho t}_{\check{D}}), \min(\widehat{\beta t}_{\check{G}}, \widehat{\beta t}_{\check{D}}), \min(\widehat{\gamma t}_{\check{G}}, \widehat{\gamma t}_{\check{D}}))$
3. $\check{G} \oplus \check{D} = (\widehat{\rho t}_{\check{G}} + \widehat{\rho t}_{\check{D}} - \widehat{\rho t}_{\check{G}}\widehat{\rho t}_{\check{D}}, -\left|\widehat{\beta t}_{\check{G}}\right|\left|\widehat{\beta t}_{\check{D}}\right|, -\left|\widehat{\gamma t}_{\check{G}}\right|\left|\widehat{\gamma t}_{\check{D}}\right|)$
4. $\check{G} \otimes \check{D} = (\widehat{\rho t}_{\check{G}}\widehat{\rho t}_{\check{D}}, -(\widehat{\beta t}_{\check{G}} + \widehat{\beta t}_{\check{D}} + \left|\widehat{\beta t}_{\check{G}}\right|\left|\widehat{\beta t}_{\check{D}}\right|), -(\widehat{\gamma t}_{\check{G}} + \widehat{\gamma t}_{\check{D}} + \left|\widehat{\gamma t}_{\check{G}}\right|\left|\widehat{\gamma t}_{\check{D}}\right|))$
5. $n\check{G} = (1 - (1 - \widehat{\rho t}_{\check{G}})^{\tilde{n}}, -\left|\widehat{\beta t}_{\check{G}}\right|^{\tilde{n}}, -\left|\widehat{\gamma t}_{\check{G}}\right|^{\tilde{n}})$

6. $\breve{G}^{\tilde{n}} = (\widehat{\rho t}^{\tilde{n}}_{\breve{G}}, -(1-(1-\left|\widehat{\beta t}_{\breve{G}}\right|)^{\tilde{n}}), -(1-(1-\left|\widehat{\gamma t}_{\breve{G}}\right|)^{\tilde{n}})$.

Definition 3.3 The score function S of $\breve{D} = (\widehat{\rho t}_{\breve{D}}, \widehat{\beta t}_{\breve{D}}, \widehat{\gamma t}_{\breve{D}})$ is evaluated as

$$S(\breve{D}) = \frac{1}{3}\left|\widehat{\rho t}_{\breve{D}} + \widehat{\beta t}_{\breve{D}} + \widehat{\gamma t}_{\breve{D}}\right| \quad S(\breve{D}) \in [0, 1]$$

As is the formulation of $\breve{D}$ accuracy function H.

$$H(\breve{D}) = \frac{1}{2}\left|\widehat{\rho t}_{\breve{D}} - \widehat{\gamma t}_{\breve{D}}\right|, H(\breve{D}) \in [0, 1]$$

if $S(\breve{D}_1) \leq S(\breve{D}_2)$ or $S(\breve{D}_1) = S(\breve{D}_2)$ but $H(\breve{D}_1) \leq H(\breve{D}_2)$ then $\breve{D}_1 \prec \breve{D}_2$ If $S(\breve{D}_1) = S(\breve{D}_2)$ and $H(\breve{D}_1) = H(\breve{D}_2)$ then $\breve{D}_1 = \breve{D}_2$ Some basic operation on Trimorphic fuzzy number.

Theorem 3.4 *Suppose* $\breve{G} = (\widehat{\rho t}_{\breve{G}}, \widehat{\beta t}_{\breve{G}}, \widehat{\gamma t}_{\breve{G}})$ *and* $\breve{D} = (\widehat{\rho t}_{\breve{D}}, \widehat{\beta t}_{\breve{D}}, \widehat{\gamma t}_{\breve{D}})$ *are two trimorphic fuzzy number and* $\tilde{n}, \tilde{n}_1 > 0$, $\tilde{n}_2 \leq 1$, *then*

1. $\breve{G} \oplus \breve{D} = \breve{D} \oplus \breve{G}$
2. $\breve{G} \otimes \breve{D} = \breve{D} \otimes \breve{G}$
3. $\tilde{n}(\breve{G} \oplus \breve{D}) = nB\breve{G} \oplus nA\breve{D}$
4. $(\breve{G} \otimes \breve{D})^{\tilde{n}} = \breve{G}^{\tilde{n}} \otimes \breve{D}^{\tilde{n}}$
5. $\tilde{n}_1\breve{G} \oplus \tilde{n}_2\breve{G} = (\tilde{n}_1 \oplus \tilde{n}_2)\breve{G}$
6. $\breve{G}^{\tilde{n}_1} \otimes \breve{G}^{\tilde{n}_2} = \breve{G}^{\tilde{n}_1+\tilde{n}_2}$

3.1 Distance in Trimorphic Fuzzy Sets

Our proposed distance formula between trimorphic fuzzy sets (TrFS) can be calculated using various distance measures. Here are some common distance formulas used for TrFS:

Definition 3.5 Let $\breve{G} = (\widehat{\rho t}_{\breve{G}}, \widehat{\beta t}_{\breve{G}}, \widehat{\gamma t}_{\breve{G}})$ and $\breve{D} = (\widehat{\rho t}_{\breve{D}}, \widehat{\beta t}_{\breve{D}}, \widehat{\gamma t}_{\breve{D}})$ be two trimorphic fuzzy numbers and $\tilde{n} \in [0, 1]$, then

1. The Hamming distance between two trimorphic fuzzy sets is $\lambda(\breve{D}, \breve{G})$

$$\lambda(\breve{D}, \breve{G}) = \sum_{\daleth=1}^{\tilde{n}} \left(\left|\widehat{\rho t}_{\breve{D}}(\bar{l}_{\daleth}) - \widehat{\rho t}_{\breve{G}}(\bar{l}_{\daleth})\right| + \left|\widehat{\beta t}_{\breve{D}}(\bar{l}_{\daleth}) - \widehat{\beta t}_{\breve{G}}(\bar{l}_{\daleth})\right| + \left|\widehat{\gamma t}_{\breve{D}}(\bar{l}_{\daleth}) - \widehat{\gamma t}_{\breve{G}}(\bar{l}_{\daleth})\right|\right)$$

Example: Let $\breve{D}_1 = (0.4, -0.3, -0.2)$, $\breve{D}_2 = (0.5, -0.4, -0.1)$ two trimorphic fuzzy numbers, the Hamming distance between these numbers is calculated as

$$\begin{aligned}\lambda(\breve{D}_1, \breve{D}_2) &= |0.4 - 0.5| + |-0.3 - (-0.4)| + |-0.2 - (-0.1)| \\ &= |-0.1| + |0.1| + |-0.1| \\ &= 0.3\end{aligned}$$

2. The normalized Hamming distance between two trimorphic fuzzy sets is $\delta(\check{D}, \check{G})$

$$\delta(\check{D}, \check{G}) = \frac{1}{n}\left(\sum_{\beth=1}^{\tilde{n}} \left(\left|\widehat{\rho t}_{\check{D}}(\bar{l}_\beth) - \widehat{\rho t}_{\check{G}}(\bar{l}_\beth)\right| + \left|\widehat{\beta t}_{\check{D}}(\bar{l}_\beth) - \widehat{\beta t}_{\check{G}}(\bar{l}_\beth)\right| + \left|\widehat{\gamma t}_{\check{D}}(\bar{l}_\beth) - \widehat{\gamma t}_{\check{G}}(\bar{l}_\beth)\right|\right)\right)$$

3. The Euclidean distance between two trimorphic fuzzy sets is $\psi(\check{D}, \check{G})$

$$\psi(\check{D}, \check{G}) = \sqrt{\sum_{\beth=1}^{\tilde{n}} \left(\left(\widehat{\rho t}_{\check{D}}(\bar{l}_\beth) - \widehat{\rho t}_{\check{G}}(\bar{l}_\beth)\right)^2 + (\widehat{\beta t}_{\check{D}}(\bar{l}_\beth) - \widehat{\beta t}_{\check{G}}(\bar{l}_\beth))^2 + (\widehat{\gamma t}_{\check{D}}(\bar{l}_\beth) - \widehat{\gamma t}_{\check{G}}(\bar{l}_\beth))^2\right)}$$

4. The of normalized Euclidean distance between two trimorphic fuzzy sets is $\sigma(\check{D}, \check{G})$

$$\sigma(\check{D}, \check{G}) = \sqrt{\frac{1}{n}\left(\sum_{\beth=1}^{\tilde{n}} \left(\left(\widehat{\rho t}_{\check{D}}(\bar{l}_\beth) - \widehat{\rho t}_{\check{G}}(\bar{l}_\beth)\right)^2 + (\widehat{\beta t}_{\check{D}}(\bar{l}_\beth) - \widehat{\beta t}_{\check{G}}(\bar{l}_\beth))^2 + (\widehat{\gamma t}_{\check{D}}(\bar{l}_\beth) - \widehat{\gamma t}_{\check{G}}(\bar{l}_\beth))^2\right)\right)}$$

3.2 Aggregation Operators for Trimorphic Fuzzy Data

Trimorphic fuzzy numbers are an expansion of conventional fuzzy numbers, which are utilized to address vulnerability and dubiousness in decision-making. In this article, we will examine a few fundamental procedures on trimorphic fuzzy numbers and their applications in different fields. Fuzzy numbers have found applications in different fields, including direction, and design. In navigation, fuzzy numbers are utilized to display dubious data and pursue informed choices in complex conditions. In risk examination, fuzzy numbers help in surveying and overseeing chances related to questionable occasions and factors. All in all, fuzzy numbers are a significant device for addressing vulnerability and dubiousness in real world. By understanding the essential procedure on fuzzy numbers and their applications in different fields, specialists, and professionals can successfully use them to resolve certifiable issues pursue more real-world problems, and make more informed decisions in uncertain environments.

In this area, we are going to be building some aggregation operators with trimorphic weighted aggregating information for example Trimorphic Weighted arithmetic operator (TrFWA) and Trimorphic Weighted Geometric (TrFWG) operator. We are looking at a number of the properties of these operators.

Definition 3.6 Let $\check{D}_\beth = (\widehat{\rho t}_\beth, \widehat{\beta t}_\beth, \widehat{\gamma t}_\beth)(\beth = 1, 2, \ldots, \tilde{n})$ be a set of TrFNs. Then trimorphic fuzzy weighted arithmetic (TrFWA) operator is defined as,

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigoplus_{\beth=1}^{\tilde{n}} \widehat{w\bar{l}}_\beth \check{D}_\beth$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ denotes the weight vector with $\check{D}_\beth(\beth = 1, 2, \ldots, \tilde{n})$ and $\sum_{\beth=1}^{\tilde{n}} \widehat{w\bar{l}}_\beth = 1, \widehat{w\bar{l}}_\beth \succ 0, \gamma \succ 0.$

3.3 Operators for Trimorphic Fuzzy Arithmetic Aggregation

To efficiently aggregate the input data, (TrFWA) and (TrFWG) operators are used. These parameters allow Trimorphic fuzzy weighted aggregation operators to capture the ambiguity and uncertainty present in real-world scenarios involving decision-making. These operators find extensive application in fields like engineering economics and medicine where decision-making is beset by imprecise and uncertain data. Trimorphic fuzzy weighted aggregation operators allow decision-makers to efficiently aggregate and assess trimorphic fuzzy number data leading to more trustworthy and accurate conclusions. Moreover, these operators meticulously and methodically aggregate trimorphic fuzzy number data advancing decision-making techniques in various domains.

Definition 3.7 Let $\check{D}_{\daleth} = (\widehat{\rho t}_{\daleth}, \widehat{\beta t}_{\daleth}, \widehat{\gamma t}_{\daleth})(\daleth = 1, 2, \ldots, \tilde{n})$ be TrFNs. In this instance, we create a trimorphic fuzzy Arithmetic Aggregation operator.

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigoplus_{\daleth=1}^{\tilde{n}} (\widehat{w\bar{l}}_{\daleth} \check{D}_{\daleth})$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ denote the weight vector with $\check{D}_{\daleth}(\daleth = 1, 2, \ldots, \tilde{n})$ and $\widehat{w\bar{l}}_{\daleth} \succ 0 \sum_{\daleth=1}^{\tilde{n}} \widehat{w\bar{l}}_{\daleth} = 1$.

Theorem 3.8 *The TrFWA operator returns a TrFN with*

$$\begin{aligned} TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) &= \bigoplus_{\daleth=1}^{\tilde{n}} (\widehat{w\bar{l}}_{\daleth} \check{D}_{\daleth}) \\ &= \left(1 - \prod_{\daleth=1}^{\tilde{n}} (1 - \widehat{\rho t}_{\daleth})^{\widehat{w\bar{l}}_{\daleth}}, -\prod_{\daleth=1}^{\tilde{n}} \left|\widehat{\beta t}_{\daleth}\right|^{\widehat{w\bar{l}}_{\daleth}}, -\prod_{\daleth=1}^{\tilde{n}} \left|\widehat{\gamma t}_{\daleth}\right|^{\widehat{w\bar{l}}_{\daleth}}\right) \end{aligned}$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ *denoted the weighted vector with* $\check{D}_{\daleth}(\daleth = 1, 2, \ldots, \tilde{n})$ *and* $\widehat{w\bar{l}}_{\daleth} \succ 0$ $\sum_{\daleth=1}^{\tilde{n}} \widehat{w\bar{l}}_{\daleth} = 1$.

Proof Mathematical induction can be used to prove this theorem.

1. Let $\tilde{n} = 2$, therefore $\widehat{w\bar{l}}_1 = 1$ for the left side of the above

$$\begin{aligned} TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) &= \widehat{w\bar{l}}_1 \check{D}_1 \oplus (\widehat{w\bar{l}}_2 \check{D}_2) \\ &= \begin{pmatrix} 1 - (1 - \widehat{\rho t}_1)^{\widehat{w\bar{l}}_1} + 1 - (1 - \widehat{\rho t}_2)^{\widehat{w\bar{l}}_2} - (1 - (1 - \widehat{\rho t}_1)^{\widehat{w\bar{l}}_1})(1 - (1 - \widehat{\rho t}_2)^{\widehat{w\bar{l}}_2}), \\ -\left|\widehat{\beta t}_1\right|^{\widehat{w\bar{l}}_1} \left|\widehat{\beta t}_2\right|^{\widehat{w\bar{l}}_2}, -\left|\widehat{\gamma t}_1\right|^{\widehat{w\bar{l}}_1} \left|\widehat{\gamma t}_2\right|^{\widehat{w\bar{l}}_2} \end{pmatrix} \\ &= (1 - (1 - \widehat{\rho t}_1)^{\widehat{w\bar{l}}_1})((1 - \widehat{\rho t}_2)^{\widehat{w\bar{l}}_2}), \widehat{\beta t}_1^{\widehat{w\bar{l}}_1} \widehat{\beta t}_2^{\widehat{w\bar{l}}_2}, -\left|\widehat{\gamma t}_1\right|^{\widehat{w\bar{l}}_1} \left|\widehat{\gamma t}_2\right|^{\widehat{w\bar{l}}_2}) \end{aligned}$$

   Thus, for $\tilde{n} = 2$, it is true.

2. Let us assume that for $\tilde{n} = k$, it is true.

$$\begin{aligned} TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) &= \bigoplus_{\daleth=1}^{k} (\widehat{w\bar{l}}_{\daleth} \check{D}_{\daleth}) \\ &= \left(1 - \prod_{\daleth=1}^{k} (1 - \widehat{\rho t}_{\daleth})^{\widehat{w\bar{l}}_{\daleth}}, -\prod_{\daleth=1}^{k} \left|\widehat{\beta t}_{\daleth}\right|^{\widehat{w\bar{l}}_{\daleth}}, -\prod_{\daleth=1}^{k} \left|\widehat{\gamma t}_{\daleth}\right|^{\widehat{w\bar{l}}_{\daleth}}\right) \end{aligned}$$

3. After that, we must demonstrate its validity for $\tilde{n} = k + 1$

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_k, \check{D}_{k+1}) = \bigoplus_{\beth=1}^{k}(\widehat{w\bar{l}_\beth}\check{D}_\beth) \oplus (\widehat{w\bar{l}_{k+1}}\check{D}_{k+1})$$

$$= \left(1 - \prod_{\beth=1}^{k}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{k}\left|\widehat{\beta}t_\beth\right|^{\widehat{w\bar{l}_\beth}}, \; -\prod_{\beth=1}^{k}\left|\widehat{\gamma}t_\beth\right|^{\widehat{w\bar{l}_\beth}}\right)$$

$$\oplus\left(1 - (1 - \widehat{\rho}t_{k+1})^{\widehat{w\bar{l}_{k+1}}}, -\left|\widehat{\beta}t_{k+1}\right|^{\widehat{w\bar{l}_{k+1}}}, \; -\left|\widehat{\gamma}t_{k+1}\right|^{\widehat{w\bar{l}_{k+1}}}\right)$$

$$= \begin{pmatrix} 1 - \prod_{\beth=1}^{k}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}} + 1 - (1 - \widehat{\rho}t_{k+1})^{\widehat{w\bar{l}_{k+1}}} - (1 - \prod_{\beth=1}^{k}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}}) \\ (1 - (1 - \widehat{\rho}t_{k+1})^{\widehat{w\bar{l}_{k+1}}}), \\ -\prod_{\beth=1}^{k+1}\left|\widehat{\beta}t_\beth\right|^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{k+1}\left|\widehat{\gamma}t_\beth\right|^{\widehat{w\bar{l}_\beth}} \end{pmatrix}$$

$$= \left(1 - \prod_{\beth=1}^{k+1}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{k+1}\left|\widehat{\beta}t_\beth\right|^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{k+1}\left|\widehat{\gamma}t_\beth\right|^{\widehat{w\bar{l}_\beth}}\right)$$

Therefore, $\tilde{n} = k + 1$ true. Hence, it is true for all n.

Example: Let $\check{D}_1 = (0.4, -0.3, -0.2)$, $\check{D}_2 = (0.5, -0.4, -0.1)$, $\check{D}_3 = (0.4, -0.3, -0.2)$ and $\check{D}_4 = (0.3, -0.2, -0.1)$ be four TrFNs and $w = (0.4, 0.3, 0.2, 0.1)$ is the weight vector of $\check{D}_\beth(1, 2, 3, 4)$ then

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_4)$$
$$= \bigoplus_{\beth=1}^{4}(\widehat{w\bar{l}_\beth}\check{D}_\beth)$$
$$= \left(1 - \prod_{\beth=1}^{4}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{4}\left|\widehat{\beta}t_\beth\right|^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{4}\left|\widehat{\gamma}t_\beth\right|^{\widehat{w\bar{l}_\beth}}\right)$$
$$= \Big(1 - (1 - \widehat{\rho}t_1)^{\widehat{w\bar{l}_1}}(1 - \widehat{\rho}t_2)^{\widehat{w\bar{l}_2}}(1 - \widehat{\rho}t_3)^{\widehat{w\bar{l}_3}}(1 - \widehat{\rho}t_4)^{\widehat{w\bar{l}_4}},$$
$$-\left|\widehat{\beta}t_1\right|^{\widehat{w\bar{l}_1}} \times \left|\widehat{\beta}t_2\right|^{\widehat{w\bar{l}_2}} \times \left|\widehat{\beta}t_3\right|^{\widehat{w\bar{l}_3}} \times \left|\widehat{\beta}t_4\right|^{\widehat{w\bar{l}_4}}, -\left|\widehat{\gamma}t_1\right|^{\widehat{w\bar{l}_1}} \times \left|\widehat{\gamma}t_2\right|^{\widehat{w\bar{l}_2}} \times \left|\widehat{\gamma}t_3\right|^{\widehat{w\bar{l}_3}} \times \left|\widehat{\gamma}t_4\right|^{\widehat{w\bar{l}_4}}\Big)$$
$$= (.4231, -.3100, -.1500)$$

Theorem 3.9 (Idempotency) *If all* $b_\beth(\beth = 1, 2, 3, \ldots, \tilde{n})$ *are equal i.e.,* $\check{D}_\beth = \check{D}$

then $\text{TrFWA}_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \check{D}$

Proof Since $\check{D}_\beth = (\widehat{\rho}t_\beth, \widehat{\beta}t_\beth, \widehat{\gamma}t_\beth)$ and $\check{D} = (\widehat{\rho}t, \widehat{\beta}t, \widehat{\gamma}t)$, then we have

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_k) = \bigoplus_{\beth=1}^{k}(\widehat{w\bar{l}_\beth}\check{D}_\beth)$$
$$= \left(1 - \prod_{\beth=1}^{\tilde{n}}(1 - \widehat{\rho}t_\beth)^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{\tilde{n}}\left|\widehat{\beta}t_\beth\right|^{\widehat{w\bar{l}_\beth}}, -\prod_{\beth=1}^{\tilde{n}}\left|\widehat{\gamma}t_\beth\right|^{\widehat{w\bar{l}_\beth}}\right)$$

$$= \left(1 - \prod_{\daleth=1}^{4}(1 - \widehat{\rho t}), -\prod_{\daleth=1}^{4}\left|\widehat{\beta t}\right|, -\prod_{\daleth=1}^{4}\left|\widehat{\gamma t}\right|\right)$$
$$= \left(1 - (1 - \widehat{\rho t}), -\left|\widehat{\beta t}\right|, -\left|\widehat{\gamma t}\right|\right)$$
$$= (\widehat{\rho t}, \widehat{\beta t}, \widehat{\gamma t}) = \check{D}$$

Theorem 3.10 (Monotonicity property) *Let* $\check{D}_\daleth = (\widehat{\rho t}_\daleth, \widehat{\beta t}_\daleth, \widehat{\gamma t}_\daleth)(\daleth = 1, 2, 3, \ldots, n)$ *and* $\check{D}'_\daleth = (\widehat{\rho t}'_\daleth, \widehat{\beta t}'_\daleth, \widehat{\gamma t}'_\daleth)$ $(\daleth = 1, 2, \ldots, n)$ *be two number of TrFNs.* $\check{D}_\daleth \leq \check{D}'_\daleth$ *for all* $\daleth = 1, 2, \ldots, n$

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_n) \leq TrFWA_{\widehat{w\bar{l}}}(\check{D}'_1, \check{D}'_2, \ldots, \check{D}'_k)$$

Theorem 3.11 (Boundedness property) *Let* $h_\daleth = (\widehat{\rho t}_\daleth, \widehat{\beta t}_\daleth, \widehat{\gamma t}_\daleth)(\daleth = 1, 2, 3 \ldots, \tilde{n})$ *be a number of TrFNs, then*

$$\check{D}^- \leq TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) \leq \check{D}^+$$

3.4 Trimorphic Fuzzy Geometric Aggregation Operators

For handling ambiguous and contradicting data in a variety of applications, trimorphic fuzzy geometric aggregation operators provide a strong and adaptable tool. These operators allow for more robust and dependable data analysis information fusion and decision-making utilizing the rich representation capabilities of trimorphic fuzzy sets and the geometric aggregation framework. More advanced and efficient aggregation techniques that fully utilize the special characteristics of trimorphic fuzzy sets are likely to emerge as this field of study develops. We will now talk about a few geometric aggregation operators that work with trimorphic fuzzy data.

Definition 3.12 Let $\check{D}_\daleth = (\widehat{\rho t}_\daleth, \widehat{\beta t}_\daleth, \widehat{\gamma t}_\daleth)(\daleth = 1, 2, \ldots, \tilde{n})$ be collection of TrFNs. Here we establish trimorphic fuzzy weighted geometric (TrFWG) operator is

$$TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigotimes_{\daleth=1}^{\tilde{n}} \check{D}_\daleth^{\widehat{w\bar{l}}_\daleth}$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ denoted the weighted vector with $\check{D}_\daleth(\daleth = 1, 2, \ldots, \tilde{n})$ and $\widehat{w\bar{l}}_\daleth \succ 0$, $\sum_{\daleth=1}^{\tilde{n}} \widehat{w\bar{l}}_\daleth = 1, \gamma \succ 0$

Theorem 3.13 *The TrFWG operator returns a TrFN with*

$$TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigotimes_{\daleth=1}^{\tilde{n}} (\check{D}_\daleth)^{\widehat{w\bar{l}}_\daleth}$$
$$= \left(\prod_{\daleth=1}^{\tilde{n}} (\widehat{\rho t}_\daleth)^{\widehat{w\bar{l}}_\daleth}, -1 + \prod_{\daleth=1}^{\tilde{n}} (1 - \left|\widehat{\beta t}_\daleth\right|)^{\widehat{w\bar{l}}_\daleth}, -1 + \prod_{\daleth=1}^{\tilde{n}} (1 - \left|\widehat{\gamma t}_\daleth\right|)^{\widehat{w\bar{l}}_\daleth}\right)$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ *denoted the weighted vector with* $\check{D}_\daleth(\daleth = 1, 2, \ldots, \tilde{n})$ *and* $\widehat{w\bar{l}}_\daleth \succ 0$, $\sum_{\daleth=1}^{\tilde{n}} \widehat{w\bar{l}}_\daleth = 1$.

Proof This theorem can be proved using mathematical induction when $\tilde{n}=2$ therefore $\widehat{wl}_1=1$ for the left side of the above

$$
\begin{aligned}
&TrFWG_{\widehat{wl}}(\check{D}_1,\check{D}_2,\ldots,\check{D}_{\tilde{n}})=(\check{D}_1)^{\widehat{wl}_1}\bigotimes(\check{D}_2)^{\widehat{wl}_2}\\
&=\begin{pmatrix}(\widehat{\rho}t_1)^{\widehat{wl}_1}(\widehat{\rho}t_2)^{\widehat{wl}_2},\\ -(2-(1-\widehat{\beta}t_1)^{\widehat{wl}_1}-(1-\widehat{\beta}t_2)^{\widehat{wl}_2})-(1-(1-\widehat{\beta}t_1)^{\widehat{wl}_1})(1-(1-\widehat{\beta}t_2)^{\widehat{wl}_2},\\ -(2-(1-\widehat{\gamma}t_1)^{\widehat{wl}_1}-(1-\widehat{\gamma}t_2)^{\widehat{wl}_2})-(1-(1-\widehat{\gamma}t_1)^{\widehat{wl}_1})(1-(1-\widehat{\gamma}t_2)^{\widehat{wl}_2})\end{pmatrix}\\
&=(\widehat{\beta}t_1^{\widehat{wl}_1}\widehat{\beta}t_2^{\widehat{wl}_2},1-(1-\widehat{\rho}t_1)^{\widehat{wl}_1})((1-\widehat{\rho}t_2)^{\widehat{wl}_2}),-1+(1-\widehat{\rho}t_1)^{\widehat{wl}_1})((1-\widehat{\rho}t_2)^{\widehat{wl}_2})\\
&=(1-(1-\widehat{\rho}t_1)^{\widehat{wl}_1})((1-\widehat{\rho}t_2)^{\widehat{wl}_2}),-\left|\widehat{\beta}t_1\right|^{\widehat{wl}_1}\left|\widehat{\beta}t_2\right|^{\widehat{wl}_2},-\left|\widehat{\gamma}t_1\right|^{\widehat{wl}_1}\left|\widehat{\gamma}t_2\right|^{\widehat{wl}_2})
\end{aligned}
$$

Therefore it is true for $\tilde{n}=2$

2. Let us assume that for $\tilde{n}=k$, it is true.

$$
\begin{aligned}
&TrFWG_{\widehat{wl}}(\check{D}_1,\check{D}_2,\ldots,\check{D}_{\tilde{n}})\\
&=\overset{k}{\underset{\beth=1}{\otimes}}(\check{D}_\beth)^{\widehat{wl}_\beth}\\
&=\left(\prod_{\beth=1}^{k}(\widehat{\rho}t_\beth)^{\widehat{wl}_\beth},-1+\prod_{\beth=1}^{k}(1-\left|\widehat{\beta}t_\beth\right|)^{\widehat{wl}_\beth},\ -1+\prod_{\beth=1}^{k}(1-\left|\widehat{\gamma}t_\beth\right|)^{\widehat{wl}_\beth}\right)
\end{aligned}
$$

3. After that, we must demonstrate its validity for $\tilde{n}=k+1$

$$
\begin{aligned}
&TrFWG_{\widehat{wl}}(\check{D}_1,\check{D}_2,\ldots,\check{D}_k,\check{D}_{k+1})\\
&=\bigotimes_{\beth=1}^{k}(\check{D}_\beth)^{\widehat{wl}_\beth}\bigotimes(\check{D}_{k+1})^{\widehat{wl}_{k+1}}\\
&=\left(\prod_{\beth=1}^{k}(\widehat{\rho}t_\beth)^{\widehat{wl}_\beth},-1+\prod_{\beth=1}^{k}(1-\left|\widehat{\beta}t_\beth\right|)^{\widehat{wl}_\beth},\ -1+\prod_{\beth=1}^{k}(1-\left|\widehat{\gamma}t_\beth\right|)^{\widehat{wl}_\beth}\right)\\
&\quad\oplus\left(\left(\widehat{\rho}t_{k+1})^{\widehat{wl}_{k+1}},-1+(1-\left|\widehat{\gamma}t_{k+1}\right|)^{\widehat{wl}_{k+1}},-1+(1-\left|\widehat{\gamma}t_{k+1}\right|\right)^{\widehat{wl}_{k+1}}\right)\\
&=\begin{pmatrix}\prod_{\beth=1}^{k+1}(\widehat{\rho}t_\beth)^{\widehat{wl}_\beth},\\ -[1-\prod_{\beth=1}^{k}(1-\widehat{\beta}t_\beth)^{\widehat{wl}_\beth}+1-(1-\widehat{\beta}t_{k+1})^{\widehat{wl}_{k+1}}-(1-\prod_{\beth=1}^{k}(1-\widehat{\beta}t_\beth)^{\widehat{wl}_\beth})\\ (1-(1-\widehat{\beta}t_{k+1})^{\widehat{wl}_{k+1})})],\\ -[1-\prod_{\beth=1}^{k}(1-\widehat{\gamma}t_\beth)^{\widehat{wl}_\beth}+1-(1-\widehat{\gamma}t_{k+1})^{\widehat{wl}_{k+1}}-(1-\prod_{\beth=1}^{k}(1-\widehat{\gamma}t_\beth)^{\widehat{wl}_\beth})\\ (1-(1-\widehat{\gamma}t_{k+1})^{\widehat{wl}_{k+1})})]\end{pmatrix}\\
&=\left(\prod_{\beth=1}^{k+1}(\widehat{\rho}t_\beth)^{\widehat{wl}_\beth},-1+\prod_{\beth=1}^{k+1}(1-\left|\widehat{\beta}t_\beth\right|)^{\widehat{wl}_\beth},\ -1+\prod_{\beth=1}^{k+1}(1-\left|\widehat{\gamma}t_\beth\right|)^{\widehat{wl}_\beth}\right)
\end{aligned}
$$

Therefore, $\tilde{n}=k+1$ true. Hence, it is true for all n. □

Theorem 3.14 (Idempotency) *If all* $\check{D}_{\beth}(\beth = 1, 2, 3, \ldots, \tilde{n})$ *are equal i.e.,* $\check{D}_{\beth} = \check{D}$ *then*

$$TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \check{D}$$

Proof Since $\check{D}_{\beth} = (\widehat{\rho t}_{\beth}, \widehat{\beta t}_{\beth}, \widehat{\gamma t}_{\beth}) = (\widehat{\rho t}, \widehat{\beta t}, \widehat{\gamma t})$, then we have

$$\begin{aligned}
TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) &= \bigotimes_{\beth=1}^{\tilde{n}} (\check{D}_{\beth})^{\widehat{w\bar{l}}_{\beth}} \\
&= \left(\prod_{\beth=1}^{\tilde{n}} (\widehat{\rho t}_{\beth})^{\widehat{w\bar{l}}_{\beth}}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\beta t}_{\beth}|)^{\widehat{w\bar{l}}_{\beth}}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\gamma t}_{\beth}|)^{\widehat{w\bar{l}}_{\beth}} \right) \\
&= \left(\prod_{\beth=1}^{\tilde{n}} (\widehat{\rho t}_{\beth})^{\widehat{w\bar{l}}_{\beth}}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\beta t}_{\beth}|)^{\widehat{w\bar{l}}_{\beth}}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\gamma t}_{\beth}|)^{\widehat{w\bar{l}}_{\beth}} \right) \\
&= \left((\widehat{\rho t}), -1 + 1 - \left|\widehat{\beta t}\right|, -1 + 1 - \left|\widehat{\gamma t}\right| \right) \\
&= (\widehat{\rho t}, \widehat{\beta t}, \widehat{\gamma t}) = \check{D}
\end{aligned}$$

Theorem 3.15 (Boundedness property) *Let* $h_{\beth} = (\widehat{\rho t}_{\beth}, \widehat{\beta t}_{\beth}, \widehat{\gamma t}_{\beth})(\beth = 1, 2, 3, \ldots, \tilde{n})$ *be a number of TrFNs, then*

$$\check{D}^{-} \leq TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) \leq \check{D}^{+}$$

Theorem 3.16 *(Monotonicity Property) Let* $\check{D}_{\beth} = (\widehat{\rho t}_{\beth}, \widehat{\beta t}_{\beth}, \widehat{\gamma t}_{\beth})(\beth = 1, 2, 3 \ldots \tilde{n})$ *and* $\check{D}'_{\beth} = (\widehat{\rho t}'_{\beth}, \widehat{\beta t}'_{\beth}, \widehat{\gamma t}'_{\beth})(\beth = 1, 2, 3 \ldots n)$ *be two number of TrFNs.* $\check{D}_{\beth} \leq \check{D}'_{\beth}$ *for all* $\beth = 1, 2, 3 \ldots \tilde{n}$

$$TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) \leq TrFWG_{\widehat{w\bar{l}}}(\check{D}'_1, \check{D}'_2 \ldots \check{D}'_{\tilde{n}})$$

4 Multiple Attribute Decision-Making Algorithm with Trimorphic Fuzzy Information

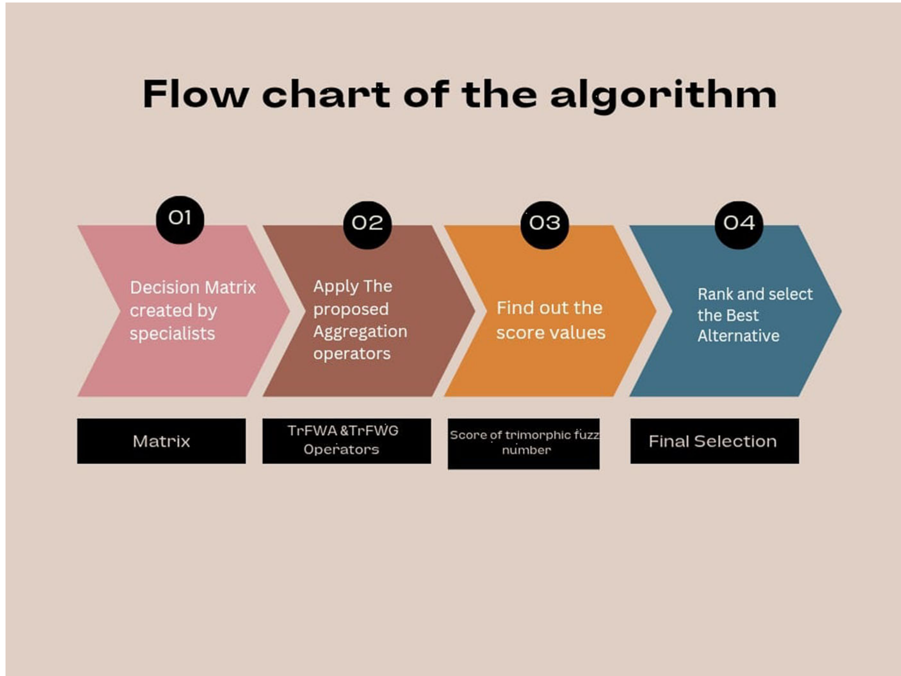
Our proposed model for a multiple attribute decision-making model using trimorphic fuzzy information based on the TrFWA (TrFWG) operators will be discussed in this section. Assume $\breve{G} = \{\breve{G}_1, \breve{G}_2, \ldots, \breve{G}_{\tilde{n}}\}$ be a discrete set of alternatives. where $G = \{G_1, G_2, \ldots, G_{\tilde{n}}\}$ is the set of criteria. Assume $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ be the weight of the criteria $G_{\beth}(\beth = 1, 2, \ldots, \tilde{n})$ such that $\widehat{w\bar{l}} \in [0, 1]$, $\sum_{\beth=1}^{\tilde{n}} \widehat{w\bar{l}}_{\beth} = 1$. Let $R = (\check{D}_{ij})_{m \times \tilde{n}} = (\widehat{\rho t}_{i \times j}, \widehat{\beta t}_{i \times j}, \widehat{\gamma t}_{i \times j})_{m \times \tilde{n}}$. be decision matrix where $\widehat{\rho t}_{ij} \in [0, 1]$, $\widehat{\beta t}_{ij} \in [-1, 0]$ and $\widehat{\gamma t}_{ij} \in [-1, 0]$. We then use the TrFWA (TrFWG) operator to construct MADM algorithm involving trimorphic fuzzy data.

The MCDM process involve following steps:

Step 1 Normalize $M_{ij} = [\widehat{\rho t}_{ij}, \widehat{\beta t}_{ij}, \widehat{\gamma t}_{ij}]_{m \times n}$, $(\breve{i} = 1, 2, \ldots, m; i = 1, 2, \ldots, n)$. Generally, the criteria can be classified into two groups, benefit criteria and cost criteria. If all the criteria are of similar type, process of normalization will not be done. But if $M_{i\breve{i}}$ contains both cost criteria and benefit criteria, then the rating values of the cost criteria can be changed into the benefit criteria by the following normalization method:

$$r_{ij} = [p_{ij}, q_{ij}, z_{ij}] = \begin{cases} M_{ij}^{c}, & \text{if the criterion is of cost type} \\ M_{ij}, & \text{if the criterion is of benefit type} \end{cases},$$

Fig. 1 Multicriteria decision-making algorithm



where $p_{ij} = \frac{\widehat{\rho}t_{ij}}{\sqrt{\sum_{i=1}^{m}\widehat{\rho}t_{ij}^2}}$, $q_{ij} = \frac{\widehat{\beta}t_{ij}}{\sqrt{\sum_{i=1}^{m}\widehat{\beta}t_{ij}^2}}$ and $z_{ij} = \frac{\widehat{\gamma}t_{ij}}{\sqrt{\sum_{i=1}^{m}\widehat{\gamma}t_{ij}^2}}$,

p_{ij}, q_{ij} ,and z_{ij} are the elements of normalized decision matrix, respectively.

Step 2 Use the proposed aggregation operators i.e., TrFWA (TrFWG) operator to calculate the aggregated TFNs for each alternatives $\breve{G}_i(i = 1, 2, \ldots, \tilde{n})$.

Step 3 Calculate the score values of all the aggregated values of the alternatives.

Step 4 Rank all the alternatives according to the score values of the alternatives. Select the alternative with the highest score as the best alternative

4.1 Flowchart

Flowcharts are a visual representation of a process or algorithm and their importance lies in their ability to break down intricate procedures into easy-to-understand steps, also provide a clear and concise visual representation of a process, reducing confusion and errors. The following flowchart helps understand the process and algorithms of models with trimorphic fuzzy information.

5 Numerical Example

In this section, we demonstrate the application of the developed approaches on a real-world example of multiple attribute decision-making problems. An organization of disabled persons wants to select the best AI-powered assistive technology (AT) for individuals with disabilities among the five alternatives ($\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$, $\hat{A}_4$ and $\hat{A}_5$), using trimorphic fuzzy set to enhance decision-making (Fig. 1).

Alternatives:

1. $\hat{A}_1$: Manual wheel chair

Table 1 Decision-maker's information

$$M = \begin{bmatrix} (0.6, -0.3, -0.1) & (0.4, -0.1, -0.6) & (0.3, -0.5, -0.4) & (0.1, -0.3, -0.2) \\ (0.4, -0.2, -0.5) & (0.1, -0.5, -0.3) & (0.5, -0.2, -0.4) & (0.5, -0.1, -0.1) \\ (0.6, -0.1, -0.1) & (0.6, -0.2, -0.2) & (0.4, -0.4, -0.1) & (0.2, -0.4, -0.5) \\ (0.2, -.3, -0.6) & (0.3, -0.4, -0.7) & (0.4, -0.5, -0.2) & (0.5, -0.4, -0.3) \\ (0.6, -0.2, -0.3) & (0.4, -0.3, -0.2) & (0.5, -0.2, -0.4) & (0.3, -0.3, -0.4) \end{bmatrix}$$

2. $\hat{A}_2$: Patriotic Wheel chair
3. $\hat{A}_3$: Standing power wheel
4. $\hat{A}_4$: AI-powered assistive technology (AT) wheel chair
5. $\hat{A}_5$: Powered wheel chair

Criteria:

1. Comfort (weight: 0.2)
2. Outdoor travel (weight: 0.1)
3. Battery life (weight:0.3)
4. Cost (weight: 0.4)

Step 1 The decision matrix is given below.
Step 2 Normalization is not needed for the data in Table 1 as all the criteria are of the same type.
Step 3 To select the best AI-powered assistive technology (AT) for individuals with disabilities from five alternatives ($\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$, $\hat{A}_4$ and $\hat{A}_5$), we utilize the (TrFWA/TrFWG) operator.
The TrFWA operator returns a TrFN with

$$TrFWA_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigoplus_{\beth=1}^{\tilde{n}} (\widehat{w\bar{l}}_\beth \check{D}_\beth)$$
$$= \left(1 - \prod_{\beth=1}^{\tilde{n}} (1 - \widehat{\rho t}_\beth)^{\widehat{w\bar{l}}_\beth}, -\prod_{\beth=1}^{\tilde{n}} \left|\widehat{\beta t}_\beth\right|^{\widehat{w\bar{l}}_\beth}, -\prod_{\beth=1}^{\tilde{n}} \left|\widehat{\gamma t}_\beth\right|^{\widehat{w\bar{l}}_\beth}\right)$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ denoted the weighted vector with $\check{D}_\beth(\beth = 1, 2, \ldots, \tilde{n})$ and $\widehat{w\bar{l}}_\beth \succ 0$ $\sum_{\beth=1}^{\tilde{n}} \widehat{w\bar{l}}_\beth = 1$.
To get $\hat{A}_1$, we have to use the following values from matrix M and weighted vector $\widehat{w\bar{l}}_1 = 0.2, \widehat{w\bar{l}}_2 = 01, \widehat{w\bar{l}}_3 = 0.3, \widehat{w\bar{l}}_4 = 0.4$
$\widehat{\rho t}_1 = 0.6, \widehat{\rho t}_2 = 0.4, \widehat{\rho t}_3 = 0.3, \widehat{\rho t}_4 = 0.1,$
$\widehat{\beta t}_1 = 0.3,\ \widehat{\beta t}_2 = 0.1,\ \widehat{\beta t}_3 = 0.5,\ \widehat{\beta t}_4 = 0.3,$
$\widehat{\gamma t}_1 = 0.1, \widehat{\gamma t}_2 = 0.6, \widehat{\gamma t}_3 = 0.4, \widehat{\gamma t}_4 = 0.2,$

$$\hat{A}_1 = \begin{pmatrix} 1 - (1 - \widehat{\rho t}_1)^{\widehat{w\bar{l}}_1} \times (1 - \widehat{\rho t}_2)^{\widehat{w\bar{l}}_2} \times (1 - \widehat{\rho t}_3)^{\widehat{w\bar{l}}_3} \times (1 - \widehat{\rho t}_4)^{\widehat{w\bar{l}}_4}, \\ -\left|\widehat{\beta t}_1\right|^{\widehat{w\bar{l}}_1} \times \left|\widehat{\beta t}_2\right|^{\widehat{w\bar{l}}_2} \times \left|\widehat{\beta t}_3\right|^{\widehat{w\bar{l}}_3} \times \left|\widehat{\beta t}_4\right|^{\widehat{w\bar{l}}_4}, \\ -\left|\widehat{\gamma t}_1\right|^{\widehat{w\bar{l}}_1} \times \left|\widehat{\gamma t}_2\right|^{\widehat{w\bar{l}}_2} \times \left|\widehat{\gamma t}_3\right|^{\widehat{w\bar{l}}_3} \times \left|\widehat{\gamma t}_4\right|^{\widehat{w\bar{l}}_4} \end{pmatrix}$$

Table 2 Score values and ranking

TrFWA
$\hat{A}_1 = (0.3185, -0.3133, -0.2392)$
$\hat{A}_2 = (0.4500, -0.1661, -0.23341)$
$\hat{A}_3 = (0.40392, -0.2828, -0.2040)$
$\hat{A}_4 = (0.3999, -0.4037, -0.3321)$
$\hat{A}_5 = (0.4428, -0.2449, -0.3523)$

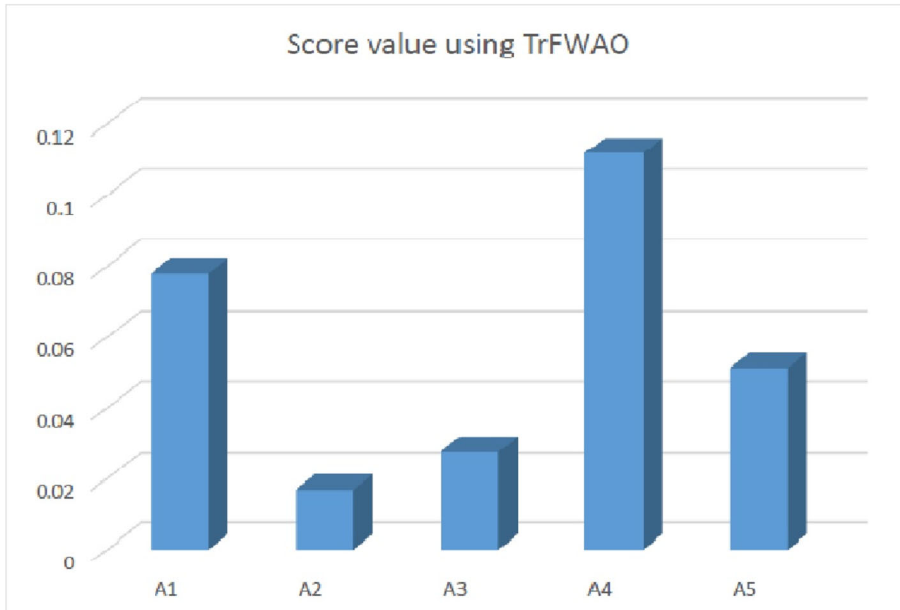


Fig. 2 Ranking of the alternatives

$$\hat{A}_1 = \begin{pmatrix} 1-(1-0.6)^{0.2}\times(1-0.4)^{0.1}\times(1-0.3)^{0.3}\times(1-0.1)^{0.4}, \\ -|0.3|^{0.2}\times|0.1|^{0.1}\times|0.5|^{0.3}\times|0.3|^{0.4}, \\ -|0.1|^{0.2}\times|0.6|^{0.1}\times|0.4|^{0.3}\times|0.2|^{0.4} \end{pmatrix}$$

$$\hat{A}_1 = (.3185, -.3133, -.2392)$$

Similarly, we can find $\hat{A}_2$, $\hat{A}_3$ and $\hat{A}_4$, we got the following Table 2.

Step 4 Determine the scores $S(\hat{A}_\daleth)$ of trimorphic fuzzy numbers $\hat{A}_\daleth$ the score function S of $\hat{A} = (\widehat{\rho t}_{\hat{A}}, \widehat{\beta t}_{\hat{A}}, \widehat{\gamma t}_{\hat{A}})$ is evaluated as.

$$S(\hat{A}) = \frac{1}{3}\left|\widehat{\rho t}_{\hat{A}} + \widehat{\beta t}_{\hat{A}} + \widehat{\gamma t}_{\hat{A}}\right| \quad S(\hat{A}) \in [0, 1]$$

$$S(\hat{A}_1) = \frac{1}{3}|0.3185 - 0.3133 - 0.2392|$$

$$S(\hat{A}_1) = 0.0780$$

$S(\hat{A}_1) = 0.0780,\ S(\hat{A}_2) = 0.0168,\ S(\hat{A}_3) = 0.0276,\ S(\hat{A}_4) = 0.112,\ S(\hat{A}_5) = 0.0515$

Step 5 Ranking all the alternatives according to the score values. $\hat{A}_4 \succ \hat{A}_1 \succ \hat{A}_5 \succ \hat{A}_3 \succ \hat{A}_2$.

A_4 is the best AI-powered assistive technology (AT) for individuals with disabilities among the five evaluated alternatives.

Table 3 Total score values and these alternatives order of ranking

TrFWG
$\hat{A}_1 = (0.2285, -0.3511, -0.2989)$
$\hat{A}_2 = (0.4070, -0.1999, -0.3090)$
$\hat{A}_3 = (0.3423, -0.3303 - 0.2968)$
$\hat{A}_4 = (0.3699, -0.4141 - 0.4014)$
$\hat{A}_5 = (0.4134, -0.2516, -0.3631)$

From graph, we can see that $\hat{A}_4$ is the first ranking, $\hat{A}_1$ is the second ranking, $\hat{A}_5$ is the third ranking $\hat{A}_3$ is the fourth and $\hat{A}_2$ is the fifth ranking (Fig. 2).

Next we have to apply TrFWG operator on the data present in decision matrix M.

The TrFWG operator returns a TrFN with

$$TrFWG_{\widehat{w\bar{l}}}(\check{D}_1, \check{D}_2, \ldots, \check{D}_{\tilde{n}}) = \bigotimes_{\beth=1}^{\tilde{n}} (\check{D}_\beth)^{\widehat{w\bar{l}}_\beth}$$
$$= \left(\prod_{\beth=1}^{\tilde{n}} (\widehat{\rho t}_\beth)^{\widehat{w\bar{l}}_\beth}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\beta t}_\beth|)^{\widehat{w\bar{l}}_\beth}, -1 + \prod_{\beth=1}^{\tilde{n}} (1 - |\widehat{\gamma t}_\beth|)^{\widehat{w\bar{l}}_\beth}\right)$$

where $\widehat{w\bar{l}} = (\widehat{w\bar{l}}_1, \widehat{w\bar{l}}_2, \ldots, \widehat{w\bar{l}}_{\tilde{n}})^T$ denoted the weighted vector with $\check{D}_\beth(\beth = 1, 2, \ldots, \tilde{n})$ and $\widehat{w\bar{l}}_\beth \succ 0$, $\sum_{\beth=1}^{\tilde{n}} \widehat{w\bar{l}}_\beth = 1$.

To get $\hat{A}_1$, we have to use the following values from matrix M and weighted vector $\widehat{w\bar{l}}_1 = 0.2$, $\widehat{w\bar{l}}_2 = 01$, $\widehat{w\bar{l}}_3 = 0.3$, $\widehat{w\bar{l}}_4 = 0.4$
$\widehat{\rho t}_1 = 0.6$, $\widehat{\rho t}_2 = 0.4$, $\widehat{\rho t}_3 = 0.3$, $\widehat{\rho t}_4 = 0.1$,
$\widehat{\beta t}_1 = 0.3$, $\widehat{\beta t}_2 = 0.1$, $\widehat{\beta t}_3 = 0.5$, $\widehat{\beta t}_4 = 0.3$,
$\widehat{\gamma t}_1 = 0.1$, $\widehat{\gamma t}_2 = 0.6$, $\widehat{\gamma t}_3 = 0.4$, $\widehat{\gamma t}_4 = 0.2$,

$$\hat{A}_1 = \begin{pmatrix} (\widehat{\rho t}_1)^{\widehat{w\bar{l}}_1} \times (\widehat{\rho t}_2)^{\widehat{w\bar{l}}_2} \times (\widehat{\rho t}_3)^{\widehat{w\bar{l}}_3} \times (\widehat{\rho t}_4)^{\widehat{w\bar{l}}_4}, \\ -1 + (1 - |\widehat{\beta t}_1|)^{\widehat{w\bar{l}}_1} \times (1 - |\widehat{\beta t}_2|)^{\widehat{w\bar{l}}_2} \times (1 - |\widehat{\beta t}_3|)^{\widehat{w\bar{l}}_3} \times (1 - |\widehat{\beta t}_4|)^{\widehat{w\bar{l}}_4}, \\ -1 + (1 - |\widehat{\gamma t}_1|)^{\widehat{w\bar{l}}_1} \times (1 - |\widehat{\gamma t}_2|)^{\widehat{w\bar{l}}_2} \times (1 - |\widehat{\gamma t}_3|)^{\widehat{w\bar{l}}_3} \times (1 - |\widehat{\gamma t}_4|)^{\widehat{w\bar{l}}_4} \end{pmatrix}$$
$$\hat{A}_1 = \begin{pmatrix} (0.6)^{0.2} \times (0.4)^{0.1} \times (0.3)^{0.3} \times (0.1)^{0.4}, \\ -1 + (1 - |-0.3|)^{0.2} \times (1 - |-0.1|)^{0.1} \times (1 - |-0.5|)^{0.3} \times (1 - |-0.3|)^{0.4}, \\ -1 + (1 - |-0.1|)^{0.2} \times (1 - |-0.6|)^{0.1} \times (1 - |-0.4|)^{0.3} \times (1 - |-0.2|)^{0.4} \end{pmatrix}$$
$$\hat{A}_1 = (0.2285, -0.3511, -0.2989)$$

Similarly, we can find $\hat{A}_2$, $\hat{A}_3$ and $\hat{A}_4$, we got the following table:

The results of which are present in Table 3.

Determine the scores $S(\hat{A}_\beth)$ of the trimorphic fuzzy numbers $\hat{A}_\beth$.

The score function S of $\hat{A} = (\widehat{\rho t}_{\hat{A}}, \widehat{\beta t}_{\hat{A}}, \widehat{\gamma t}_{\hat{A}})$ is evaluated as

$$S(\hat{A}) = \frac{1}{3}\left|\widehat{\rho t}_{\hat{A}} + \widehat{\beta t}_{\hat{A}} + \widehat{\gamma t}_{\hat{A}}\right| \quad S(\hat{A}) \in [0, 1]$$

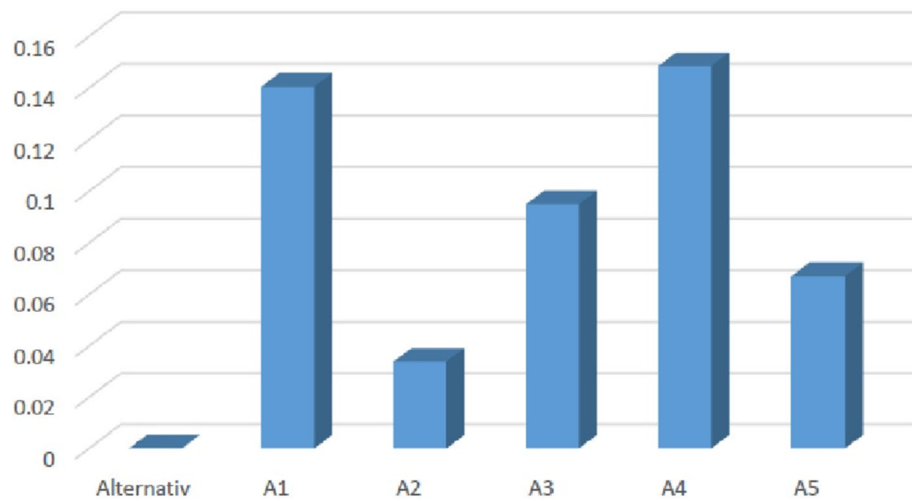


Fig. 3 Ranking of the alternatives

$$S(\hat{A}_1) = \frac{1}{3}|0.2285 - 0.3511 - 0.2989|$$
$$S(\hat{A}_1) = 0.14051$$

$S(\hat{A}_1) = 0.14051,\ S(\hat{A}_2) = 0.0339,\ S(\hat{A}_3) = 0.0949,\ S(\hat{A}_4) = 0.1485,\ S(\hat{A}_5) = 0.0671$

Ranking order

$\hat{A}_4 \succ \hat{A}_1 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_2.$

A_4 is the best AI-powered assistive technology (AT) for individuals with disabilities among five evaluated alternatives (Fig. 3).

The graph indicates that $\hat{A}_4$ is in the top position followed by $\hat{A}_1$ in the second place $\hat{A}_3$ in the third $\hat{A}_5$ in the fourth and $\hat{A}_2$ in the fifth. The ranking results of the alternatives by both the operators are given below:

Operators	Ranking order
TrFWA	$\hat{A}_4 \succ \hat{A}_1 \succ \hat{A}_5 \succ \hat{A}_3 \succ \hat{A}_2$
TrFWG	$\hat{A}_4 \succ \hat{A}_1 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_2$

5.1 Comparison Analysis

This section will cover a comparison between other known aggregation operators and the suggested aggregation operators under trimorphic fuzzy information. The suggested aggregation operators were contrasted with picture fuzzy weighted averaging/ geometric (PFWA/PFWG) [6], picture fuzzy order weighted averaging/ geometric (PFOWA/PFOWG) [6], picture fuzzy Dombi weighted averaging/ geometric (PFDWA/PFDWG) [11], picture fuzzy Dombi order weighted averaging/ geometric (PFDOWA/PFDOWG) [11], picture fuzzy Dombi hybrid weighted averaging/ geometric (PFDHWA/PFDHWG) [11], picture fuzzy Einstein weighted averaging/ geometric (PFEWA/PFEWG), picture fuzzy Einstein order weighted averaging/ geometric (PFEOWA/PFEOWG) [12]. The following in Table 4 is a summary of our findings.

We can see from the table above that the ranking of the existing operators PFWA [6] and PFWG [6] operators are $\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$ and $\hat{A}_2 \succ \hat{A}_3 \succ \hat{A}_4 \succ \hat{A}_5 \succ \hat{A}_1$ respectively in which the choice of second alternative is confusing similarly since the ranking results of other existing aggregation operators are not similar, even there is confusion in the selection of the best alternative among them. In the case of our proposed trimorphic

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Table 4 Ranking order of the alternatives

Operators	Ranking
PFWA [6]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFWG [6]	$\hat{A}_2 \succ \hat{A}_3 \succ \hat{A}_4 \succ \hat{A}_5 \succ \hat{A}_1$
PFOWA [6]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFOWG [6]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFEWA [12]	$\hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_2 \succ \hat{A}_5 \succ \hat{A}_1$
PFEWG [12]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFEOWA [12]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFEOWG [12]	$\hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_1$
PFDWA [11]	$\hat{A}_2 \succ \hat{A}_1 \succ \hat{A}_5 \succ \hat{A}_4 \succ \hat{A}_3$
PFDWG [11]	$\hat{A}_2 \succ \hat{A}_3 \succ \hat{A}_4 \succ \hat{A}_5 \succ \hat{A}_1$
PFDOWA [11]	$\hat{A}_2 \succ \hat{A}_1 \succ \hat{A}_5 \succ \hat{A}_3 \succ \hat{A}_4$
PFDOWG [11]	$\hat{A}_3 \succ \hat{A}_2 \succ \hat{A}_4 \succ \hat{A}_5 \succ \hat{A}_1$
PFDHWA [11]	$\hat{A}_2 \succ \hat{A}_1 \succ \hat{A}_5 \succ \hat{A}_4 \succ \hat{A}_3$
PFDHWG [11]	$\hat{A}_1 \succ \hat{A}_3 \succ \hat{A}_5 \succ \hat{A}_5 \succ \hat{A}_4$

fuzzy aggregation operators, there is no confusion as compared to other existing aggregation operators with regard to their choice of the first and also of second option.

Therefore, it is more reliable and efficient to use our proposed trimorphic fuzzy aggregation operators as opposed to those of the past.

6 TOPSIS Method

Hwang and Yoon [9] developed the TOPSIS method. It is a very useful MCDM technique. This method is based on the idea of the level of optimality established in an alternative, where different criteria stand in for the idea of the best option [27]. The foundation of the TOPSIS approach is the notion that, in actual practice, the best alternative will be the closest to the positive ideal solution and the farthest from the negative ideal solution.The TOPSIS technique has been used in a variety of decision scenarios. This is because (a) it is computationally feasible, (b) it is significant in solving various viable decision issues, (c) it is simple, and (d) it is easy to understand. TOPSIS looks at these extremes in an attempt to find the option that minimizes the adverse effect and most closely matches the desired criteria [16]. When presented with a variety of options and competing evaluation criteria, this method offers a methodical and structured way to decide. TOPSIS helps decision-makers make well-informed decisions by giving equal weight to the advantages and disadvantages of each alternative. Because it enables a thorough evaluation that considers the relative importance of each criterion this method is especially helpful in scenarios where there are trade-offs between various criteria [17]. In general, TOPSIS provides a methodical and orderly framework for making decisions assisting in making sure the selected option is appropriate for fulfilling the given requirements and goals.

Flowchart of TOPSIS Method

Using a variety of criteria the TOPSIS method offers a methodical way to assess and rank options. This method is a useful tool for decision-making in a variety of fields because it consists of a set of clearly defined steps that aid in selecting the best option from a set of options (Fig. 4).

The Algorithm of TOPSIS method is given below.

TOPSIS method has the below steps:

Fig. 4 Algorithm of TOPSIS method

Step 1 Construct the decision matrix.

$$M_{ij} = [\widehat{\rho t}_{ij}, \widehat{\beta t}_{ij}, \widehat{\gamma t}_{ij}]_{m \times n}$$

Step 2 Normalize $M_{ij} = [\widehat{\rho t}_{ij}, \widehat{\beta t}_{ij}, \widehat{\gamma t}_{ij}]_{m \times n}$, $(\breve{i} = 1, 2, \ldots, m; i = 1, 2, \ldots, n)$. Generally, the criteria can be classified into two groups, benefit criteria and cost criteria. If all the criteria are of similar type, process of normalization will not be done. But if $M_{i\breve{i}}$ contains both cost criteria and benefit criteria, then the rating values of the cost criteria can be changed into the benefit criteria by the following normalization method:

$$r_{ij} = [p_{ij}, q_{ij}, z_{ij}] = \begin{cases} M_{ij}^{c}, & \text{if the criterion is of cost type} \\ M_{ij}, & \text{if the criterion is of benefit type} \end{cases},$$

where $p_{ij} = \frac{\widehat{\rho t}_{ij}}{\sqrt{\sum_{i=1}^{m} \widehat{\rho t}_{ij}^{2}}}$, $q_{ij} = \frac{\widehat{\beta t}_{ij}}{\sqrt{\sum_{i=1}^{m} \widehat{\beta t}_{ij}^{2}}}$ and $z_{ij} = \frac{\widehat{\gamma t}_{ij}}{\sqrt{\sum_{i=1}^{m} \widehat{\gamma t}_{ij}^{2}}}$,
p_{ij}, q_{ij}, and z_{ij} are the elements of normalized decision matrix respectively.

Step 3 Weighted normalized decision matrix i.e.,

$$M_{ij} = G_{ij} \widehat{w\bar{l}}_{j} \forall i, j$$

where the weight given to attribute j is denoted by $\widehat{w\bar{l}}_{j}$.

Step 4 Finding the positive-ideal $\left(\check{D}^{+}\right)$ and negative-ideal $\left(\check{D}^{-}\right)$ solutions i.e.,

$$\check{D}^{+} = \left\{\left\{\left(\max_{q} \widehat{\rho t}_{pq}(\bar{l})/p \in R\right), \left(\min_{q} \widehat{\beta t}_{pq}(\bar{l})/p \in R^{/}\right), \left(\min_{q} \widehat{\gamma t}_{pq}(\bar{l})/p \in R^{/}\right)\right\}\right\}$$
$$\check{D}^{-} = \left\{\left\{\left(\min_{q} \widehat{\rho t}_{pq}(\bar{l})/p \in R\right), \left(\max_{q} \widehat{\beta t}_{pq}(\bar{l})/p \in R^{/}\right), \left(\max_{q} \widehat{\gamma t}_{pq}(\bar{l})/p \in R^{/}\right)\right\}\right\}$$

where R and R' are associated with benefit and cost criteria respectively. Calculate Hamming distance between each element of decision matrix with a positive ideal $\left(\check{D}^{+}\right)$ and negative-ideal $\left(\check{D}^{-}\right)$ of separation measure. Positive ideal values are calculated by the below formula:

$$\lambda(\check{D}, \check{G})_{i}^{+} = \sum_{\beth=1}^{\tilde{n}} \left(\left|\widehat{\rho t}_{ij}(\bar{l}_{\beth}) - \widehat{\rho t}_{i}^{+}(\bar{l}_{\beth})\right| + \left|\widehat{\beta t}_{ij}(\bar{l}_{\beth}) - \widehat{\beta t}_{i}^{+}(\bar{l}_{\beth})\right| + \left|\widehat{\gamma t}_{ij}(\bar{l}_{\beth}) - \widehat{\gamma t}_{i}^{+}(\bar{l}_{\beth})\right|\right).$$

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Negative ideal values are calculated by the below formula:

$$\lambda(\check{D}, \check{G})_i^- = \sum_{\beth=1}^{\tilde{n}} \left(\left|\widehat{\rho t}_{ij}\left(\bar{l}\beth\right) - \widehat{\rho t}_i^-\left(\bar{l}\beth\right)\right| + \left|\widehat{\beta t}_{ij}\left(\bar{l}\beth\right) - \widehat{\beta t}_i^-\left(\bar{l}\beth\right)\right| + \left|\widehat{\gamma t}_{ij}\left(\bar{l}\beth\right) - \widehat{\gamma t}_i^-\left(\bar{l}\beth\right)\right| \right).$$

Step 5 Find the closeness coefficient. Closeness coefficient ranks options according to how close they are to the optimal solution. This is determined by dividing the distance of an alternative from the NIS by the total of its distances from the PIS and the NIS. Mathematically, it is expressed as

$$F_i^+ = \frac{\lambda(\check{D}, \check{G})_i^+}{\lambda(\check{D}, \check{G})_i^+ + \lambda(\check{D}, \check{G})_i^-}$$

Step 6 We will select the best alternative on the basis of closeness to the positive ideal solution. The option with the highest closeness coefficient is regarded as the best. This methodical approach guarantees a fair assessment of each option in light of the selection criteria.

6.1 Numerical Example

A multi-criteria approach to decision-making that can be used in a variety of situations including hiring decisions is called TOPSIS. A healthcare organization seeks to identify the most suitable AI-powered assistive technology to improve patient care. The organization has the following available choices:

Choices/Alternatives:
$\hat{F}_1$: System A
$\hat{F}_2$: System B
$\hat{F}_3$: System C
$\hat{F}_4$: System D
Criteria:
C_1 : Accuracy (weight: 0.3)
C_2 : Ease of use (weight: 0.2)
C_3 : Scalability (weight: 0.1)
C_4 : Cost (weight: 0.4).

The fourth criteria are non-beneficial while the first three are beneficial. The four alternatives are to be assessed by the decision-makers under the four attributes whose weighting vector is $\widehat{w\bar{l}} = (0.3, 0.2, 0.1, 0.4)$. The decision matrices of the three experts are given below.

For the selection of suitable system, we use our suggested methodologies. The step-by-step details are given below:

Step 2 Create a decision matrix.

Expert 1

$$M_1 = \begin{bmatrix} & C_1 & C_2 & C_3 & C_4 \\ \hat{F}_1 & (.5, -.4, -.1) & (.3, -.4, -.1) & (.2, -.2, -.2) & (.3, -.1, -.4) \\ \hat{F}_2 & (.3, -.2, -.3) & (.5, -.4, -.1) & (.3, -.2, -.1) & (.1, -.1, -.4) \\ \hat{F}_3 & (.5, -.5, -.2) & (.5, -.2, -.1) & (.2, -.2, -.3) & (.5, -.2, -.1) \\ \hat{F}_4 & (.6, -.2, -.3) & (.5, -.4, -.1) & (.2, -.2, -.1) & (.2, -.5, -.1) \end{bmatrix}$$

Expert 2

$$M_2 = \begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hat{F}_1 & (.5,-.4,-.3) & (.3,-.1,-.1) & (.1,-.2,-.4) & (.1,-.2,-.4) \\ \hat{F}_2 & (.3,-.2,-.3) & (.5,-.4,-.4) & (.5,-.2,-.1) & (.5,-.1,-.1) \\ \hat{F}_3 & (.5,-.5,-.2) & (.5,-.2,-.2) & (.3,-.1,-.1) & (.5,-.2,-.1) \\ \hat{F}_4 & (.6,-.2,-.3) & (.5,-.4,-.1) & (.5,-.1,-.4) & (.1,-.5,-.4) \end{array}$$

Expert 3

$$M_3 = \begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hat{F}_1 & (.1,-.4,-.1) & (.3,-.4,-.3) & (.5,-.1,-.4) & (.5,-.1,-.4) \\ \hat{F}_2 & (.3,-.2,-.3) & (.5,-.1,-.1) & (.5,-.5,-.1) & (.5,-.2,-.4) \\ \hat{F}_3 & (.5,-.5,-.2) & (.1,-.1,-.5) & (.3,-.1,-.1) & (.1,-.2,-.1) \\ \hat{F}_4 & (.3,-.2,-.3) & (.5,-.4,-.1) & (.1,-.1,-.4) & (.1,-.5,-.1) \end{array}$$

Combining the Tables 1, 2 and 3 and finding the average of each corresponding element of all tables, we get

$$M_4 = \begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hat{F}_1 & (.37,-.40,-.17) & (.30,-.30,-.17) & (.27,-.17,-.33) & (.30,-.13,-.40) \\ \hat{F}_2 & (.30,-.20,-.30) & (.50,-.30,-.20) & (.43,-.30,-.10) & (.37,-.13,-.30) \\ \hat{F}_3 & (.50,-.50,-.20) & (.37,-.17,-.27) & (.27,-.13,-.17) & (.37,-.20,-.10) \\ \hat{F}_4 & (.50,-.20,-.30 & (.50-.40,-.10) & (.27,-.13,-.30) & (.13,-.50,-.20) \end{array}$$

Step 3 We normalized decision matrix M_4 which is given as M_5.

$$M_5 = \begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hat{F}_1 & (.44,-.57,-.34) & (.35,-.49,-.44) & (.44,-.43,-.67) & (.48,-.23,-.73) \\ \hat{F}_2 & (.35,-.29,-.60) & (.59,-.49,-.51) & (.69,-.75,-.20) & (.60,-.23,-.55) \\ \hat{F}_3 & (.59,-.71,-.40) & (.44,-.28,-.69) & (.44,-.33,-.35) & (.60,-.35,-.18) \\ \hat{F}_4 & (.59,-.29,-.60) & (.59,-.66,-.26) & (.44,-.33,-.61) & (.21,-.88,-.36) \end{array}$$

Step 4 Construction of weighted normalized decision matrix:

$$M_6 = \begin{array}{c|cccc} & C_1(.3) & C_2(.2) & C_3(.1) & C_4(.4) \\ \hat{F}_1 & (.13,-.17,-.10) & (.07,-.10,-.09) & (.04,-.04,-.07) & (.19,-.09,-.29) \\ \hat{F}_2 & (.11,-.09,-.18) & (.12,-.10,-.10) & (.07,-.08,-.02) & (.24,-.09,-.22) \\ \hat{F}_3 & (.18,-.21,-.12) & (.09,-.06,-.14) & (.04,-.03,-.04) & (.24,-.14,-.07) \\ \hat{F}_4 & (.18,-.09,-.18) & (.12,-.13,-.05) & (.04,-.03,-.06) & (.08,-.35,-.14) \end{array}$$

Step 5 Determination of ideal $\left(\check{D}^{+}\right)$ and negative-ideal $\left(\check{D}^{-}\right)$solutions.

The first three are beneficial and last one is non-beneficial using maximum and minimum operations, respectively.
The positive ideal solution is
$\check{D}^{+} = \{(.18, -.21, -.18), (.12, -.13, -.14)(.07, -.08, -.07), (.08, -.09, -.07)\}$
The negative ideal solution is
$\check{D}^{-} = \{(.11, -.09, -.1), (.07, -.06, -.05)(.04, -.03, -.02), (.24, -.35, -.29)\}$

This step uses hamming distance operators to calculate the separation between each normalized decision matrix element and the positive and negative ideal solutions.

$\lambda(\check{D}, \hat{F})_i^{+} = \{.70, .62, .48, .63\}$ and $\lambda(\check{D}, \hat{F})_i^{-} = \{.55, .63, .77, .62\}$

Table 5 Values of closeness coefficients

Overall rating values
$\hat{F}_1 = 0.560$
$\hat{F}_2 = 0.496$
$\hat{F}_3 = 0.384$
$\hat{F}_4 = 0.504$

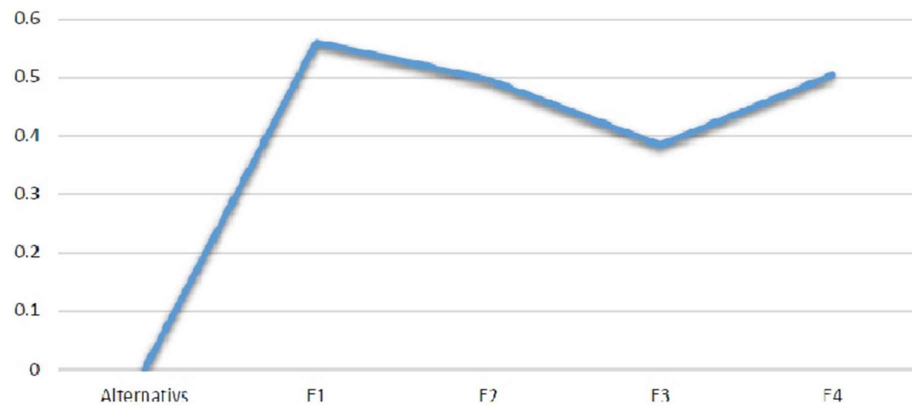

Fig. 5 Comparison analysis

Table 6 Ranking order of alternatives

Operators	Ranking
PFWA [6]	$\hat{F}_1 > \hat{F}_4 > \hat{F}_2 > \hat{F}_3$
PFWG [6]	$\hat{F}_1 > \hat{F}_2 > \hat{F}_4 > \hat{F}_3$
PFOWA [6]	$\hat{F}_1 > \hat{F}_4 > \hat{F}_2 > \hat{F}_3$
PFOWG [6]	$\hat{F}_4 > \hat{F}_1 > \hat{F}_3 > \hat{F}_2$
PFDWA [11]	$\hat{F}_1 > \hat{F}_2 > \hat{F}_4 > \hat{F}_3$
PFDWG [11]	$\hat{F}_4 > \hat{F}_1 > \hat{F}_2 > \hat{F}_3$
PFDOWA [11]	$\hat{F}_1 > \hat{F}_2 > \hat{F}_4 > \hat{F}_3$
PFDOWG [11]	$\hat{F}_4 > \hat{F}_1 > \hat{F}_2 > \hat{F}_3$

Step 6 Find the closeness coefficient. The values of closeness coefficients are given in Table 5.
Step 7 Rank all the alternatives according to the closeness coefficients.

$\hat{F}_1 > \hat{F}_4 > \hat{F}_2 > \hat{F}_3$.
$\hat{F}_1$ is the best system among all the evaluated alternatives (Fig. 5).

From the graph, we see that $\hat{F}_1$ is the first ranking, $\hat{F}_4$ is the second ranking, $\hat{F}_2$ is the third ranking, and $\hat{F}_3$ is the fourth.

6.2 Comparison Analysis

This section presents the comparison between other known aggregation operators and the results of TOPSIS method under trimorphic fuzzy information. The results of TOPSIS method were contrasted with picture fuzzy weighted averaging/geometric (PFWA/PFWG) [6], picture fuzzy order weighted averaging/geometric (PFOWA/PFOWG) [6], picture fuzzy Dombi weighted averaging/geometric (PFDWA/PFDWG) [11], picture fuzzy Dombi order weighted averaging/ geometric (PFDOWA/PFDOWG) [11]. The summary of all the ranking of the outcomes is given in Table 6.

We can see from the table above that the ranking of the existing operators PFWA [6] and PFWG [6] operators is $\hat{F}_1 > \hat{F}_4 > \hat{F}_2 > \hat{F}_3$ and $\hat{F}_1 > \hat{F}_2 > \hat{F}_4 > \hat{F}_3$, respectively, in which the choice of second alternative is confusing similarly since the ranking results of other existing aggregation operators are not similar even when there is confusion in the selection of the best alternative among them. In the case of our proposed trimorphic fuzzy TOPSIS method, there is no confusion as compared to other existing techniques with regard to their choice of the first and also of the second option.

7 Conclusion

A novel idea of trimorphic fuzzy sets, which are extensions of the current fuzzy sets, has been introduced. For managing ambiguity in real-world problems, TrFS is a more useful tool. It resolved the issues encountered by traditional fuzzy sets, intuitionistic fuzzy sets, bipolar fuzzy sets, and picture fuzzy sets, particularly when applied to hiring promotions and the selection of appropriate AI-powered assistive technology for people with disabilities. It's an innovative idea that provides effective representation for problems involving imprecision uncertainty and decision-making. We have outlined some aggregation operators for trimorphic fuzzy information, covered some fundamental ideas in brief, and created a model of multiple attribute decision-making in a trimorphic environment. By comparing suggested aggregation operators with other existing techniques, we utilize trimorphic fuzzy aggregation operators to build the MADM approach for the selection of AI-powered assistive technology for people with disabilities. We have developed the extended form of TOPSIS method with TrFNs. To validate the developed approach and demonstrate its effectiveness and practicality, this paper presents a practical example for evaluating and selecting AI-powered assistive technologies to support individuals with disabilities. After comparing the outcomes of our suggested techniques with those of the current approaches, we discovered that our results were clearer and more accurate.

In future work, our proposed models will be applied to various fields, such as engineering, hydro power plants, renewable energy, and mobile robots, in our upcoming study.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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