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ALGORITHM FOR FINDING CONNECTED RESOLVING NUMBER OF A GRAPH

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Abstract

For an ordered subset $\beta = \{\beta_1, \beta_2, ..., \beta_k\}$ of vertices and v in a connected graph $G = (V, E)$, the k-vector

$$r(v|\beta) = (d(v, \beta_1), d(v, \beta_2), d(v, \beta_k))$$

is the metric representation of vertex v with respect to β. β is a resolving set for G if various vertices of G have different representations with respect to β. A minimum resolving set is the lowest cardinality resolving set and $\dim(G)$ is the cardinality of the dimension of G. A resolving set B of G is connected if the subgraph $\overline{B}$ produced by B is a nontrivial connected subgraph of G. The cardinality of the minimal resolving set is the metric dimension of G, while the cardinality of the lowest connected resolving set is the connected metric dimension of G. A connected metric dimension of G, denoted by $\mathrm{cdim}(G)$, is the lowest cardinality of a connected resolving set. The connected resolving number of a graph can be found using the algorithm presented in this work.

1. Introduction

Graphs considered are simple, connected, undirected, with a lot of edges but no loop. The study of connected resolving number is the fastest growing area in graph theory because of its numerous and varied applications in fields like algorithmic designs, communications networks, social sciences, and other areas.

The challenge of figuring out a graph's metric dimension was discussed by Harary and Melter [1]. Slater [2] discussed the use of this concept to long-range navigational aids. Melter and Tomescu [3] investigated the metric dimension problem of grid graphs. Khuller et al. [4] have studied the metric dimension problem for multi-dimensional grids and trees. Batiha et al. [5] studied the connected metric dimension types of ladder graphs, namely, ladder, circular, open, and triangular ladder graphs, as well as open diagonal and slanting ladder graphs. The metric dimension of the Jahangir graph J_{2n}, as well as the partition and connected dimension of the wheel graph W_n, were calculated by Tomescu et al. [6]. Paths on n vertices constitute a family of graphs with constant metric dimension since Chartrand et al. demonstrated in [7] that a graph G has metric dimension 1 if and only if $G = P_n$. According to Javaid et al. in [8], the planar graph antiprism A_n is a family of regular graphs with a constant metric dimension, such that for any $n \geq 5$, $\dim(A_n) = 3$. Ahmad et al. [9] calculated the metric dimension of $P(n, 2) \odot K_1$. Kurniawati et al. [10] determined the resolving domination number of friendship graphs and its operation. Muhammad and Susilowati [11] proposed the computer program for determining the basis and dimension of a graph. In [12], the connected metric dimension of path graph P_n, cycle graph C_n, wheel graph W_n, star graph $K_{1,n-1}$, and complete graph K_n is investigated. It is shown that the connected metric dimension of cycle graph C_n, $n \geq 3$ is 2, wheel graph W_n, $n \geq 7$ is $\left\lfloor \frac{2n+2}{5} \right\rfloor + 1$, star graph $K_{1,n-1}$, $n \geq 4$ is $n-1$, complete graph K_n, $n \geq 3$ is $n-1$ and path graph P_n, $n \geq 2$ is 2. In [13], it is shown that the connected metric dimension at a vertex of tree T is 1 if v is an end vertex and 2 if v is not an end vertex, Petersen graph P is 4, and wheel graph W_n, $n \geq 7$ is $\left\lfloor \frac{2n+2}{5} \right\rfloor + 1$. For more results, see [14-22].

In Section 2, we introduce the basic concepts. In Section 3, we present an algorithm for finding the connected resolving number of a graph. Finally, Section 4 presents the conclusion of this paper.

2. Preliminaries

Lemma 2.1 [23]**.** *Let G be a connected graph and $S \subseteq V(G)$. If S contains a resolving set of G, then S is a resolving set of G.*

Proposition 2.2. *For a variety of well-known graph types, this proposition shows some results obtained from $cdim(G)$* [24].

(1) *For triangular book graph T_n with n vertices, $cdim(T_n) = n - 2$.*

(2) *For quadrilateral book graph B_n^4 with n vertices, $cdim(B_n^4) = \frac{n}{2}$.*

(3) *For knots graph K_n with n vertices, $cdim(K_n) = 3$.*

(4) *For crystal planar map C_n with n vertices and k blocks, $cdim(C_n) = 3$.*

3. Algorithm to Determine Connected Resolving Set of a Graph

The algorithm consists of two steps: first, it determines the induced subgraph from a connected graph, and second, it determines the resolving set for the graph's connected subgraph.

3.1. An algorithm to determine the connection between each pair of unique vertices

Using a as the initial vertex and b as the destination vertex, the algorithm's first step is to determine the relationship between two vertices, a and b, in G.

After initializing all singleton subsets $B = \{\{x\} | x \in V(G)\}$, we examine the vertex, and if we find a disconnection with other vertices, the procedure is terminated. Otherwise, we construct the new set

$$B = \{\{x_1, x_2\} | x_1, x_2 \in V(G)\}.$$

If two of the vertices in $V(G)$ are connected, the process will be continued until we get $B = \{\{x_1, x_2, ..., x_j\} | x_i \in V(G), 1 \leq i \leq j\}$, where there are no two connected vertices in $V(G)$. Then we declare $\{x_1, x_2, ..., x_j\}$ as connected subgraph G.

3.2. An algorithm for identifying a connected resolving set

When defining a collection of subsets of $V(G)$, say

$$B = \{\{x_1, x_2\} | x_1, x_2 \in V(G)\},$$

we examine how each connected vertex is represented in relation to a subset of $V(G)$. The process is finished if no two vertices in $V(G)$ have the same representation with regard to a subset of $V(G)$. Given a subset of $V(G)$, if two vertices in $V(G)$ will have the same representation, then construct the new set $B = \{\{x_1, x_2, x_3\} | x_1, x_2, x_3 \in V(G)\}$, the process will be continued until we get

$$B = \{\{x_1, x_2, ..., x_j\} | x_i \in V(G), 1 \leq i \leq j\},$$

while, in $V(G)$, no two vertices have the same representation with respect to $\{x_1, x_2, ..., x_j\}$. Then we declare $\{x_1, x_2, ..., x_j\}$ as connected resolving set.

Algorithm 1. Check connect pseudo code

Function Check Connect of set of vertices (Adjacency matrix, Chosen Points Representation, X)

1. $X \leftarrow 1$
2. $r \leftarrow X$
3. $C \leftarrow 0$
4. **for** $i \leftarrow 0$ to length$(r)-1$ do
5. **for** $j \leftarrow i+1$ to length (r) do
6. **if** $A(r(i), r(j)) = 1$ then $C(i) \leftarrow C(i)+1$
7. **end**
8. **end**
9. **end**
10. $FS = C > 0$
11. Out $= \prod FS$

Algorithm 2. Pseudo code to determine connected resolving set

Function Check a set of vertices is a resolving set $(G,\ X)$

$S \leftarrow \sum X$

$n \leftarrow$ number of points in graph

$X \leftarrow 1$

subset $\leftarrow X$

for $i \leftarrow$ length(subset)

for $j \leftarrow$ length(n)

Resolving Set $(i,\ j) = D_{min} | P_{X(i)=1} \rightarrow P_{G(j)} |$

End for

End for

for $i \leftarrow 1$ to length(Resolving Set)-1

for $j \leftarrow i + 1$ to length(Resolving Set)

if Resolving Set$(:,\ j) =$ Resolving Set$(:,\ i)$ then output $= 0$

else

output $= 1$

end if

end for

end for

4. Conclusion

Numerous fields, including image processing, combinatorial optimization, wireless sensor network localization, robot navigation, network discovery and verification, and image processing, have utilized the connected metric dimension. The connected resolving number of a graph can be found using the algorithm we proposed in this work.

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